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EXPERIMENTAL RESPONSE FUNCTIONS AND RESPONSE MATRICES FOR 2.54-CM X 2.54-CM AND 7.62-CM X 7.62-CM BISMUTH GERMANATE SCINTILLATION DETECTORS

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Experimental Response Functions and Response Matrices for 2.54-cm x 2.54-cm and 7.62-cm x 7.62-cm Bismuth Germanate Scintillation Detectors

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We report experimental response functions at twelve gamma-ray energies over the range of 123.6 keV to 11.67 MeV for a 2.54-cm-diameter x 2.54-cm-long and a 7.62-cm-diameter x 7.62-cm long bismuth germanate scintillation detector. The measurements were made at, or corrected to correspond to, source-to-detector distances of 100 cm. The resolutions of the detectors at a gamma-ray energy of 661.6 keV are 11.6 percent for the small detector and 13.0 percent for the large detector. We also present an interpolation method for generating a response function at any gamma-ray energy in the range of 123.6 keV to 11.67 MeV for either scintillator using the experimental response functions. Additionally, this method is used in constructing response matrices for unfolding gamma-ray pulse-height distributions acquired with the detectors. A computer code written in FORTRAN-77 generates the response matrices.

25 Response function; Bismuth Germanate; Scintillator; Response matrix; Gamma-Ray detector

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CONTENTS

I. INTRODUCTION

As part of an extensive experimental study of the energy and angular distributions of neutrons and gamma rays produced from 50- to 200-MeV proton and deuteron bombardment of thick elemental targets, we acquired two sizes of bismuth germanate (BGO) inorganic scintillators, 2.54-cm-diameter x 2.54-cm-long and 7.62-cm-diameter x 7.62-cm-long, for the measurement of gamma-ray energy spectra. The gamma-ray energies of interest ranged from approximately 250 keV to 15 MeV with the most intense part of each gamma-ray spectrum lying below E_γ = 4500 keV (Refs. 1-3). The neutron energy spectra are dominated by neutrons with $E_n \leq 10$ MeV (Refs. 2-6).

BGO was chosen instead of NaI(Tl) because of its favorable characteristics regarding detection of high-energy gamma rays in the presence of both large numbers of low-energy gamma rays and large numbers of neutrons with energies primarily less than 10 MeV. In one of the early investigations of the gamma-ray response of a BGO detector, Evans (Ref. 7) found the ratio of the photopeak efficiency of a 3.8-cm x 3.8-cm BGO scintillator to that of the same size NaI(Tl) scintillator to be 3.3, 4.5, and 5.6 at E_γ = 662, 1332, and 2754 keV, respectively. He concludes that because of the increased photopeak efficiency and the relative flatness of the BGO photopeak efficiency curve as compared to NaI(Tl), BGO detectors are the best choice for measurement of high-energy gamma rays accompanied by large

numbers of low-energy gamma rays. Further studies of BGO detectors (Refs. 8-10) show that the neutron sensitivity of a 7.62-cm x 7.62-cm BGO scintillator is considerably less than that of the same size NaI(Tl) detector. Hausser, *et al* (Ref. 9) show that the BGO to NaI(Tl) neutron sensitivity ratio is approximately 0.2 at E_n = 500 keV, 1 at E_n = 3500 keV, and 1.2 at $E_n \geq$ 5000 keV. The authors of Ref. 9 predict that this ratio is at least less than 0.1 for $E_n <$ 500 keV and conclude that BGO as compared to NaI(Tl) has a much greater gamma-ray-to-neutron detection ratio. The only disadvantage of a BGO scintillator, as compared to NaI(Tl) discussed in the studies, is the poorer energy resolution of BGO (Refs. 1 and 4). However, in our work, energy resolution of the gamma-ray detector is not important since the gamma rays produced from medium-energy charged particle bombardment of targets primarily form a continuum (Refs. 2 and 3).

In order to obtain gamma-ray energy spectra using a BGO detector, the detector response must be deconvoluted from the measured gamma-ray pulse-height distributions. The deconvolution process is commonly performed with response matrix unfolding computer codes such as the widely used FERDO code (Ref. 11). The response matrix input to the unfolding code consists of numerous BGO response functions spanning the gamma-ray energy range of interest. A response function is the pulse-height distribution resulting from a monoenergetic gamma-ray source of unit intensity incident on

the BGO scintillator. Each response function can be quite complicated in form and depends on many factors: the quality of the BGO scintillation crystal, the photomultiplier tube and associated detector electronics, the gamma-ray source and detector configuration, and the detector shielding.

The standard practice for obtaining a set of response functions is to calculate or measure them for a BGO detector in a specific experimental arrangement. The calculations are performed with Monte Carlo electron-photon transport codes such as CYLTRAN (Ref. 12) and PEP-C (Ref. 13). Although these codes produce photopeak efficiencies which agree well with measured efficiencies for different sized BGO detectors (Refs. 13 and 14), there is no extensive comparison between calculated and measured response functions. Numerous measurements of photopeak efficiencies, total efficiencies, resolutions, multigamma responses, and response functions for 2.54-cm x 2.54-cm and 7.62-cm x 7.62-cm BGO detectors exist (Refs. 8 and 10, 13-17). However, the only complete set of response functions for a single BGO detector and gamma-ray source configuration is the result of the measurements of Moss, *et al* (Refs. 16 and 17) for a 7.62-cm x 7.62-cm detector positioned 30 cm from various gamma-ray point sources. These response functions are 17-parameter analytical functions fitted to some of the measured pulse-height distributions. For our work we found these theoretical response functions to be inadequate,

especially at the higher gamma-ray energies, because they do not include several effects: (1) the energy dependence of the detector resolution within a given response, (2) the asymmetries of the photopeak, single-escape peak, and double-escape peak due to secondary radiation and charged particle escape from the crystal, (3) the variation in shape of the Compton distribution with increasing gamma-ray energy, and (4) in-scattering of gamma-rays from the detector housing materials and surrounding shielding materials. These effects vary in importance depending on the BGO detector and the experimental arrangement. We were primarily concerned with the first three.

Because of the inadequacies of the calculated and measured response functions of BGO scintillators reported in the literature, we measured the response functions for a 2.54-cm x 2.54-cm and a 7.62-cm x 7.62-cm BGO detector at twelve gamma-ray energies between 123.6 keV and 11.67 MeV. This paper reports these measurements along with tabulations and graphs of intrinsic photopeak efficiencies and detector energy resolutions. Furthermore, we present a computer code written in FORTRAN-77 which uses these measured response functions to generate a response matrix for use in unfolding codes. The resultant response matrix can be used for unfolding gamma-ray pulse-height distributions over the energy range of approximately 600 keV to 12 MeV. Both the reported response functions and response matrix generation code should be of general use because they can be used for

other BGO detectors of the same size with equivalent or poorer energy resolution merely by "smearing" these response functions with the appropriate resolution factor.

II. RESPONSE FUNCTION MEASUREMENT DETAILS

BGO Detectors

We used 2.54-cm x 2.54-cm and 7.62-cm x 7.62-cm BGO detectors manufactured by Bicron Corporation. Each detector came as an integral assembly with photomultiplier tube and magnetic/light shield. A schematic diagram of each detector is shown in Fig. 1 and Fig. 2. Not all of the packaging details of the scintillators are shown in the figures. Further details can be obtained from Bicron Corporation by specifying the model numbers: 2.54-cm x 2.54-cm, Model # 1M1BGO/1.5; 7.62-cm x 7.62-cm, Model # 3M3BGO/3.

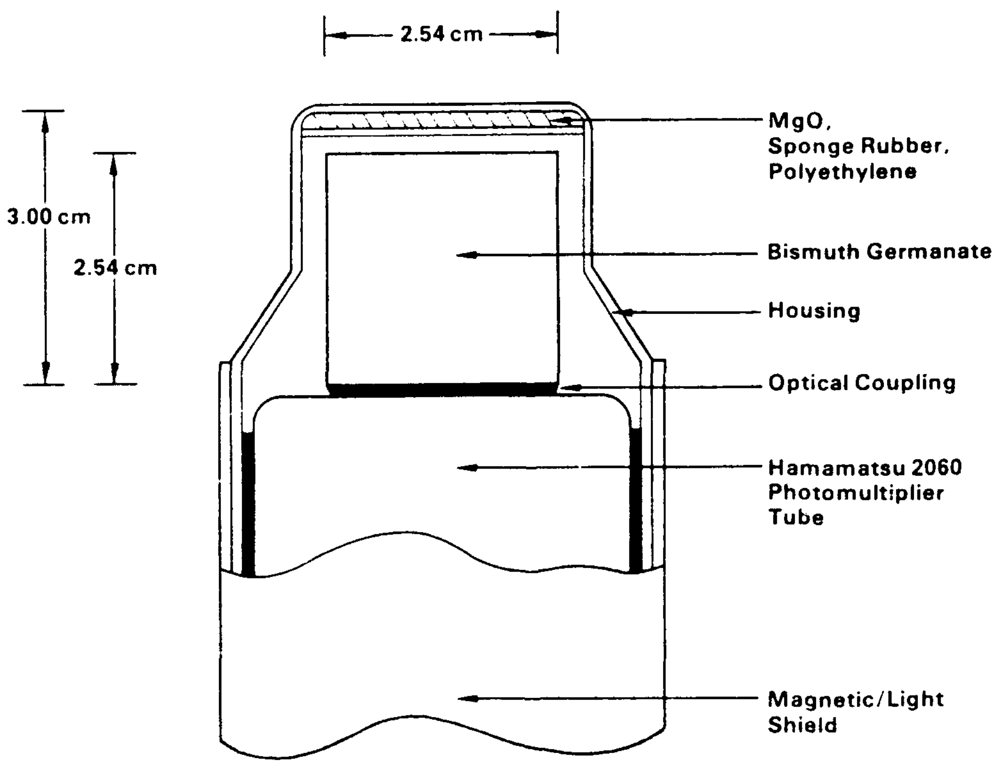

Figure 1. Schematic of the 2.54-cm-diameter x 2.54-cm-long BGO scintillation detector. (Schematic shown with permision of Bicron Corporation).

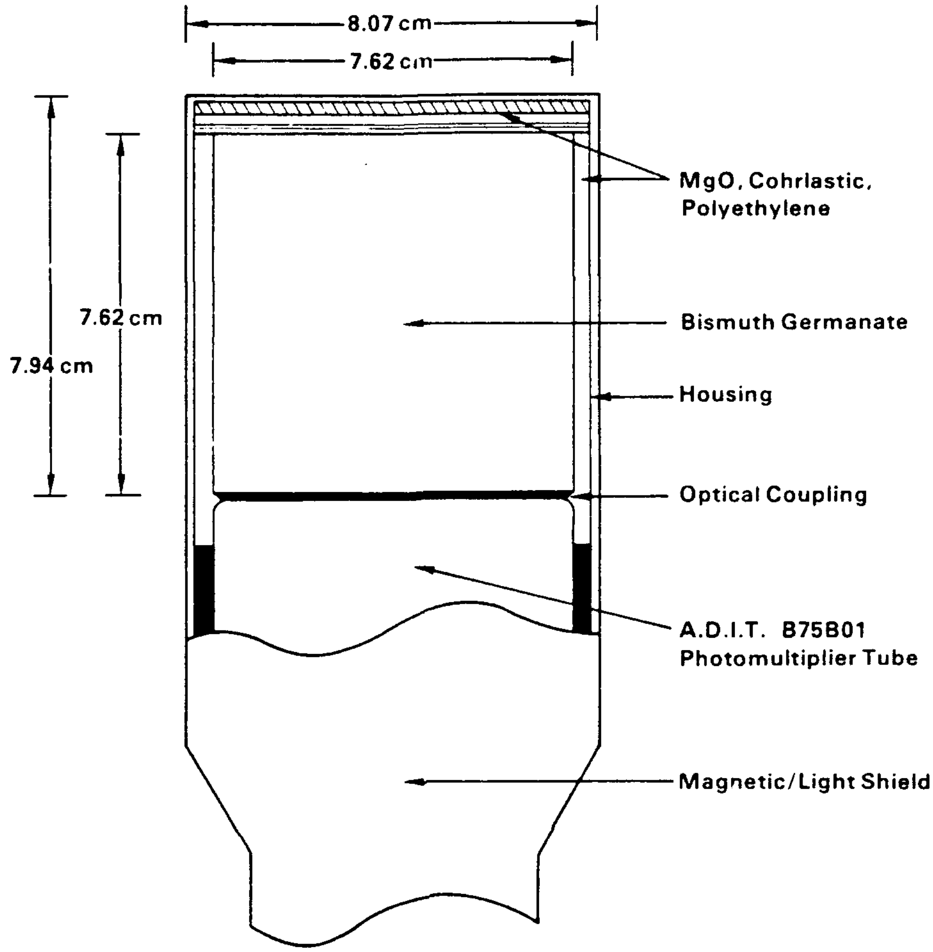

Figure 2. Schematic of the 7.62-cm-diameter x 7.62-cm-long BGO scintillation detector. (Schematic shown with permission of Bicron Corporation).

Gamma-Ray Sources

For the response function measurements we used eight standard gamma-ray point sources of known intensity spanning the energy range of 123.6 to 1836 keV (Table 1). We also used four sources of gamma rays of unknown intensity from the $^{23}Na(n,\gamma)^{24}Na$, $^{9}Be(\alpha,n\gamma)^{12}C$, $^{19}F(p,\alpha\gamma)^{16}O$, and $^{11}B(p,\gamma)^{12}C$ reactions spanning the energy range of 2754 keV

Table 1. Gamma-Ray Sources Used For BGO Scintillation Detector Response Function Measurements.

Nuclide or Reaction	Gamma-Ray Energies (keV)
^{57}Co (a)	122, 136.5 }123.6
^{203}Hg (a)	279.2
^{85}Sr (a)	514
^{137}Cs (a)	661.6
^{54}Mn (a)	834.8
^{65}Zn (a)	511, 1115.5
^{22}Na (a)	511, 1274.5
^{88}Y (a)	898, 1836
$^{23}Na(n,\gamma)^{24}Na$ (b)	1368.6, 2754
$^{9}Be(\alpha,n\gamma)^{12}C$ (c)	others < 1000, 2223, 4440
$^{19}F(p,\alpha\gamma)^{16}O$ (d)	511, 6140, 6920, 7120
$^{11}B(p,\gamma)^{12}C$ (d)	511, 4440, 11,670

(a) Calibrated point source purchased from Isotope Products Laboratory.
(b) Uncalibrated extended source produced by neutron activation.
(c) Uncalibrated extended $^{238}Pu/Be$ neutron/gamma-ray source.
(d) Uncalibrated extended source produced by radiative proton capture.

to 11.67 MeV (Table 1). The point sources had a nominal strength of 100 μCi and were of a thin, "scatterless" disc-type construction (Ref. 18). The uncertainty in intensity for each source was certified to at most $\pm$ 4.6 percent. Each of the gamma-ray sources produced from neutron, alpha, or proton reactions was an extended source (Table 2). The reaction $^{23}Na(n,\gamma)^{24}Na$ was produced using sodium metal coated with paraffin and bombarded with thermal neutrons

Table 2. Summary Of Extended Gamma-Ray Sources Dimensions.

Nuclide Designation and Reaction	Source Description and Dimensions
^{24}Na $^{23}Na(n,\gamma)^{24}Na$	2.5-cm x 2.5-cm square, 1 cm thick
$^{238}Pu/Be$ $^{9}Be(\alpha,n\gamma)^{12}C$	0.8-cm-diameter x 3-cm-long cylinder in large water tank [a]
$^{16}O^*$ $^{19}F(p,\alpha\gamma)^{16}O$	2-cm-diameter x 0.4-cm-thick disk
$^{12}C^*$ $^{11}B(p,\gamma)^{12}C$	1.2-cm-diameter x 0.2-cm-thick disk

[a] The large water tank served as a neutron moderator.

from the United States Air Force Academy (USAFA) 200-keV Cockroft-Walton accelerator/neutron generator. Both the $^{19}F(p,\alpha\gamma)^{16}O$ and $^{11}B(p,\gamma)^{12}C$ reactions were produced using the USAFA 400-keV Van de Graaff accelerator with proton energies somewhat greater than the resonance energies at E_p = 340 keV and 163 keV, respectively. The targets consisted of pressure-packed powder discs of sodium fluoride and boron.

Experimental Arrangement

For the response function measurements, the eight point sources were on the detector axis line a distance of 100 cm from the front face of each BGO scintillator. The choice of this distance was a compromise between the data acquisition rate driven by source intensity and the desire to have a

nearly parallel beam of gamma rays incident on the detector. The center of the cylindrical ^{238}Pu/Be source was also 100 cm from the front face of the BGO detector. However, between the source and detector, 10 cm from the end of the ^{238}Pu/Be cylinder, we placed a 3.81-cm-diameter cylindrical plug consisting of a 3-cm-thick piece of polyethylene, a 0.9-cm-thick piece of lead, and a 4-cm-thick piece of polyethylene (Fig. 3). This plug reduced the number of neutrons and low-energy gamma-rays from the source. The

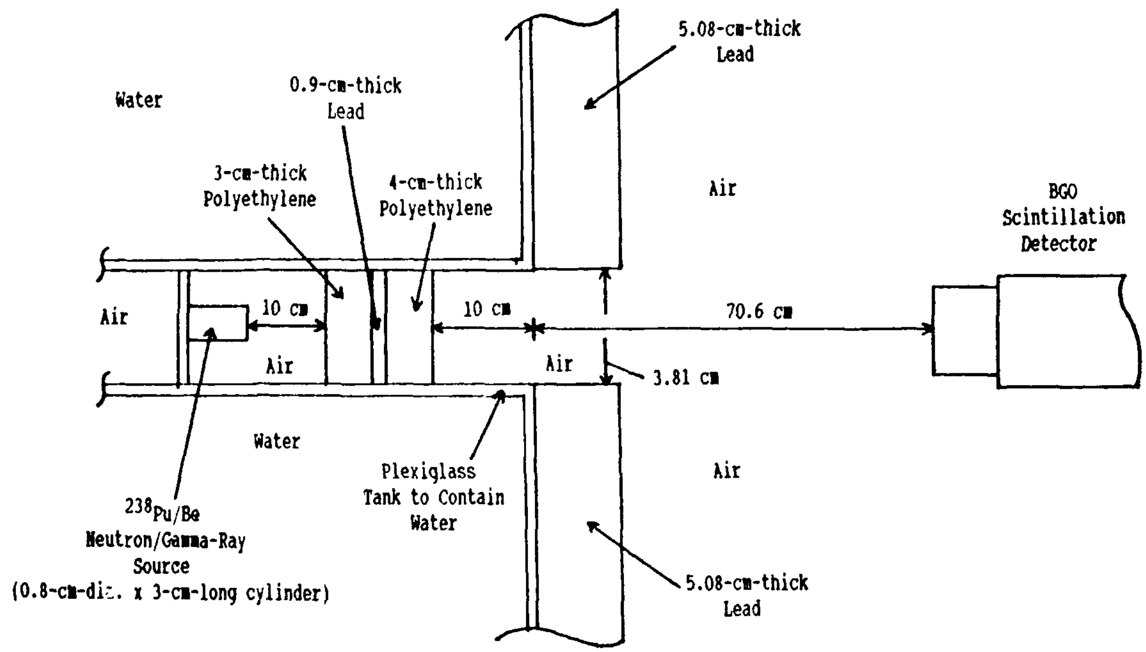

Figure 3. Experimental arrangement for the BGO response function measurements using the ^{238}Pu/Be neutron/gamma-ray source.

outside of the water tank facing the detector was lined with 5.08 cm of lead to reduce the number of 2223-keV capture gamma rays escaping from the tank. The center of the ^{24}Na source was located 5 cm from the front face of the 2.54-cm x 2.54-cm BGO detector and 10 cm from the front face of the 7.62-cm x 7.62-cm detector due to the extremely low intensity of the ^{24}Na source. The centers of the sodium fluoride and boron source targets were 34 cm from the front faces of the detectors. The targets were positioned at the middle of an evacuated 52.1-cm-diameter aluminum scattering chamber with thin, Plexiglass viewing ports (Fig. 4).

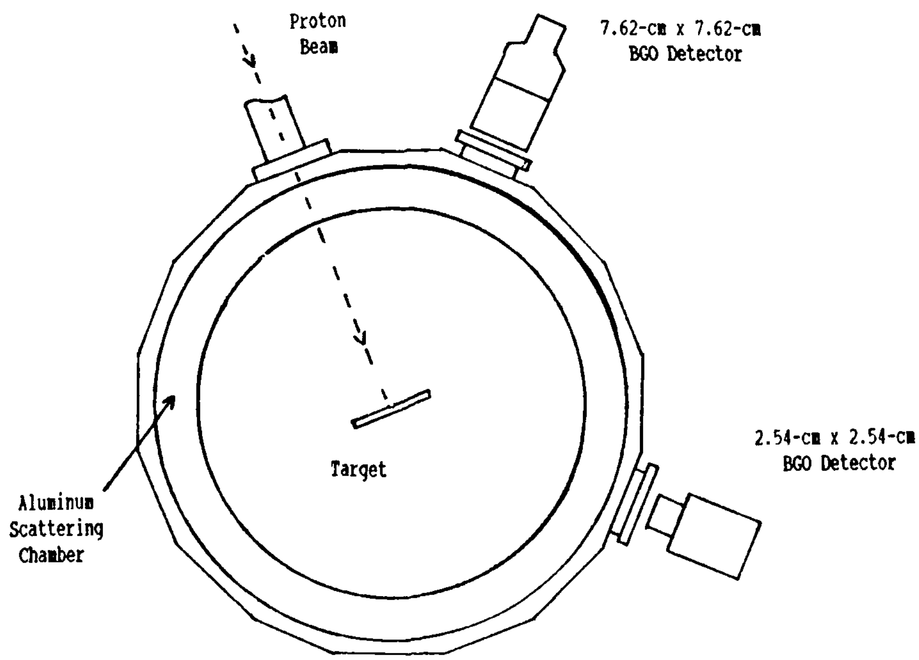

Figure 4. Experimental arrangement for the BGO response function measurements using the sodium fluoride and boron source targets.

Again, this distance was chosen due to the low intensity of the sources. Since the response function for a scintillation detector is known to vary with source-to-detector distance (Ref. 19), the ^{88}Y point source was also used for response function measurements at distances of 5 cm and 25 cm for the 2.54-cm x 2.54-cm BGO detector and 10 cm and 50 cm for the 7.62-cm x 7.62-cm BGO detector. These measurements along with further information from the literature was used to modify those measurements taken with source-to-detector distances less than 100 cm in order to make them correspond to 100-cm source-to-detector distances. This modification is described in the analysis section of the paper. A summary of the source-to-detector distances for the response function measurements is presented in Table 3.

Gamma-ray scattering effects were minimized for as many of the response function measurements as possible. Both the detectors and sources were a minimum of 100 cm from the floor, ceiling, and nearest wall. The eight manufactured sources along with the ^{24}Na source were suspended with tape from the ceiling, hanging 150 cm above the floor. Both BGO detectors were mounted on aluminum camera tripods 150 cm above the floor. The scattering effects were larger for the remaining sources since the ^{238}Pu/Be source was contained in a water tank and the accelerator-produced sources were targets mounted in an aluminum scattering chamber. However,

Table 3. Summary Of Gamma-Ray Source-To-Detector Distances For The 2.54-cm x 2.54-cm And 7.62-cm x 7.62-cm BGO Scintillation Detectors.

Nuclide or Reaction	Source-to-Detector Distance[a] 2.54-cm x 2.54-cm (cm)	Source-to-Detector Distance[a] 7.62-cm x 7.62-cm (cm)
^{57}Co	100	100
^{203}Hg	100	100
^{85}Sr	100	100
^{137}Cs	100	100
^{54}Mn	100	100
^{65}Zn	100	100
^{22}Na	100	100
^{88}Y	5, 25, 100	10, 50, 100
$^{23}Na(n,\gamma)^{24}Na$	5	10
$^{9}Be(\alpha,n\gamma)^{12}C$	100	100
$^{19}F(p,\alpha\gamma)^{16}O$	34	33.7
$^{11}B(p,\gamma)^{12}C$	34	33.7

[a] All distances are measured from the center of the gamma-ray source to the front face of the BGO scintillation detector.

the detectors remained mounted on aluminum camera tripods for these measurements.

We simultaneously acquired pulse-height distributions for both the 2.54-cm x 2.54-cm and 7.62-cm x 7.62-cm BGO detectors for each gamma-ray source. The dynode signals from the photomultiplier tube bases were fed through ORTEC 113 preamplifiers into ORTEC 450 or 572 amplifiers. The resulting amplified signals were digitized into 1024

channels with Nuclear Data 65 and 66 multichannel analyzers. For each pulse-height distribution measurement, we adjusted the amplifier gains to maximize the 1024-channel digitization. The pulse-height distribution acquisition times ranged from 1 to 5.5 hours. Before and after each measurement, we performed an energy calibration of the pulse-height axis using 4 to 8 known gamma-ray photopeaks. There were no noticeable gain drifts during any of the data runs. As soon as possible after a data run, we acquired background pulse-height distributions by removing the source from the laboratory room. For most of the measurements, the background run times were equal to or longer than the data run times. Furthermore, the pulse-height axes of the background distributions were energy calibrated separately from the data runs. Both the data and background 1024-channel distributions along with the energy calibration spectra were stored in ASCII format for analysis using a personal computer and a MicroVAX.

III. ANALYSIS

The first step in converting the measured pulse-height distributions into response functions was energy calibration of the pulse-height axis of each data and background file. We fitted each peak of a calibration spectrum with a Gaussian function plus a second-order polynomial function using the generalized function minimization program MINUIT (Ref. 20). For closely-spaced peaks, simultaneous fits involving multiple Gaussian functions plus one second-order polynomial function were used. The result of these fits were specific energy values corresponding to channel numbers. We then performed a linear least-squares fit of the channel number and energy data to determine the pulse-height axis calibration.

Next we subtracted the background distribution from the data distribution channel by channel. However, since the pulse-height calibrations for corresponding data and background files were usually slightly different, we first rebinned the background distribution into the appropriate data distribution bin values and widths. This process was visually checked for consistency using peaks in the data and background distributions at 1460 and 2615 keV due to the radioactive nuclides ^{40}K and ^{208}Tl. These nuclides are found in ordinary construction materials such as those in the walls of our laboratory.

For the measurements using the ^{57}Co, ^{203}Hg, ^{85}Sr, ^{137}Cs, ^{54}Mn, and ^{16}O* sources, the only further analysis required to determine the response functions was photopeak area determination and subsequent normalization of each pulse-height distribution to one incident photon. However, the remaining sources emit gamma rays corresponding to more than one energy, or secondary reactions produce various energy gamma rays. (See Table 1.) Actually, the ^{57}Co and ^{16}O* sources produce multiple-energy gamma rays. But the two photopeaks at 122 and 136.5 keV in the ^{57}Co pulse-height distribution are not resolved because of the BGO resolution and are treated as a single-energy gamma ray corresponding to 123.6 keV. This energy was computed using a weighted average based on the relative intensities for the 122 and 136.5 keV gamma rays. The 6920- and 7120-keV gamma rays emitted from ^{16}O* can be neglected because of their extremely low intensities--less than 3 percent of the 6140-keV gamma-ray intensity based on our measurements. The 511-keV contribution is discussed later.

For the multiple-energy sources, the pulse-height distributions due to the low-energy gamma rays must be subtracted from the overall multiple gamma-ray distribution in order to obtain a response function. The procedure was as follows. We used the ^{65}Zn source to obtain the pulse-height distribution due only to 1115.5-keV gamma rays by subtracting the distribution resulting from the 511-keV gamma-rays. The subtraction process used the ^{85}Sr

distribution, which results from monoenergetic 514-keV gamma rays. We multiplied the slope and intercept of the ^{85}Sr pulse-height axis by the ratio 511/514 so that the 514-keV distribution corresponded to the 511-keV distribution along the pulse-height axis. The appropriate scale factor to match the number of counts in the 511- and 514-keV photopeaks was determined iteratively by viewing the Compton region of the resultant 1115.5-keV pulse-height distribution. This Compton region is flat when the 511-keV distribution is scaled appropriately. This same procedure was followed for the ^{22}Na and ^{88}Y multigamma distributions. However, for ^{88}Y the 834.8-keV spectrum from ^{54}Mn was used in the subtraction process.

For the ^{24}Na source, the 1368.6-keV piece of the pulse-height distribution was eliminated using the resultant 1274.5-keV spectrum from ^{22}Na. However, a simple scaling of the slope and intercept of the 1274.5-keV spectrum to approximate the 1368.6-keV distribution could not be used. Since the Compton region of a pulse-height distribution is relatively flat and structureless for low-energy gamma rays, we stretched the middle of this region for 1274.5 keV so that the 1274.5-keV photopeak was centered on the channel number corresponding to 1368.6 keV. Furthermore, this technique preserved the position of the backscatter peak which varies little with gamma-ray energy for energies greater than approximately 1000 keV. An analogous process was followed to eliminate the 2223-keV piece of the $^{238}Pu/Be$

pulse-height distribution using the resultant 2754-keV spectrum and contracting the Compton region instead of stretching it.

The only other significant extraneous pieces in the $^{238}Pu/Be$ distribution to be removed were contributions from 511- and 842-keV gamma rays. We determined this from looking at the $^{238}Pu/Be$ pulse-height distribution and also comparing with the $^{16}O^*$ distribution in the low pulse-height regions. The 842-keV contribution was easily removed using the ^{54}Mn distribution in the manner described previously. However, the 511-keV piece could not be removed because it is the result of annihilation radiation from both the extended source and the detector assembly materials and materials of the water tank and shielding housing the $^{238}Pu/Be$ source. Likewise, the 511-keV contributions to the $^{16}O^*$ and $^{12}C^*$ pulse-height distributions could not be eliminated. Therefore, the response functions for gamma-ray energies of 4440 keV, 6140 keV, and 11.67 MeV should not be used for deconvolution of measured gamma-ray spectra in pulse-height regions below approximately 600 keV. This is not a significant limitation in their use, however, since most gamma-ray spectra are complicated in this region by annihilation radiation and useful information is difficult to extract.

Lastly, for those pulse-height distributions with low statistics and fairly large contributions from extraneous gamma rays, we applied a five-point data smoothing procedure

to that part of the distribution affected by the subtraction process. Figure 5 shows the pulse-height distributions taken with the 2.54-cm x 2.54-cm BGO detector from the ^{65}Zn, ^{88}Y, ^{24}Na, and ^{238}Pu/Be multiple gamma-ray sources and the distributions after subtraction of the lower-energy gamma-ray contributions and subsequent smoothing. Figure 6 shows the equivalent pulse-height distributions for the 7.62-cm x 7.62-cm BGO detector.

A further correction was applied to the 2754-keV spectrum, resulting from ^{24}Na, to correct for shape variations with source-to-detector distance. Figure 7 shows the unnormalized pulse-height distributions from ^{88}Y acquired with the 2.54-cm x 2.54-cm detector at source-to-detector distances of 5, 25, and 100 cm. The greatest shape differences occur between the 5- and 100-cm spectra in the pulse-height region below approximately 700 keV. This is also true for the ^{88}Y spectra acquired with the 7.62-cm x 7.62-cm BGO detector. Therefore, the 2754-keV distribution was modified below 700 keV using the 1836-keV distribution taken at 100 cm. Due to the small differences between the ^{88}Y spectra for 25- and 100-cm source-to-detector distances, we did not correct the $^{16}O^*$ and $^{12}C^*$ distributions since they were taken at source-to-detector distances of 34 cm.

The final step in obtaining the response functions was to normalize the single-energy pulse-height distributions to one gamma ray incident on the detector. For the gamma-ray sources of known intensities, this was straightforward,

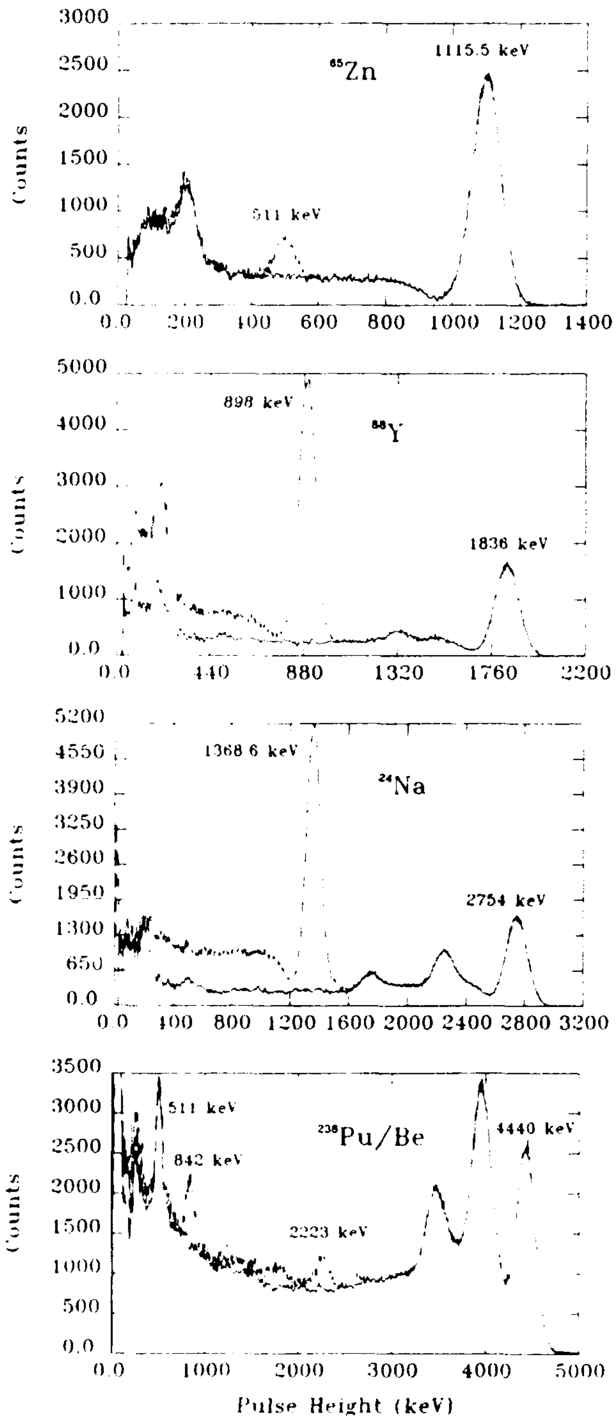

Figure 5. Pulse-height distributions acquired with a 2.54-cm x 2.54-cm BGO detector for gamma rays from ^{65}Zn, ^{88}Y, ^{24}Na, and $^{238}Pu/Be$ radioactive sources. The dashed curves include the effects of all gamma-rays emitted from the sources. The solid curves are the distributions from only the highest energy gamma rays emitted from the sources. The pulse-height distributions are not normalized.

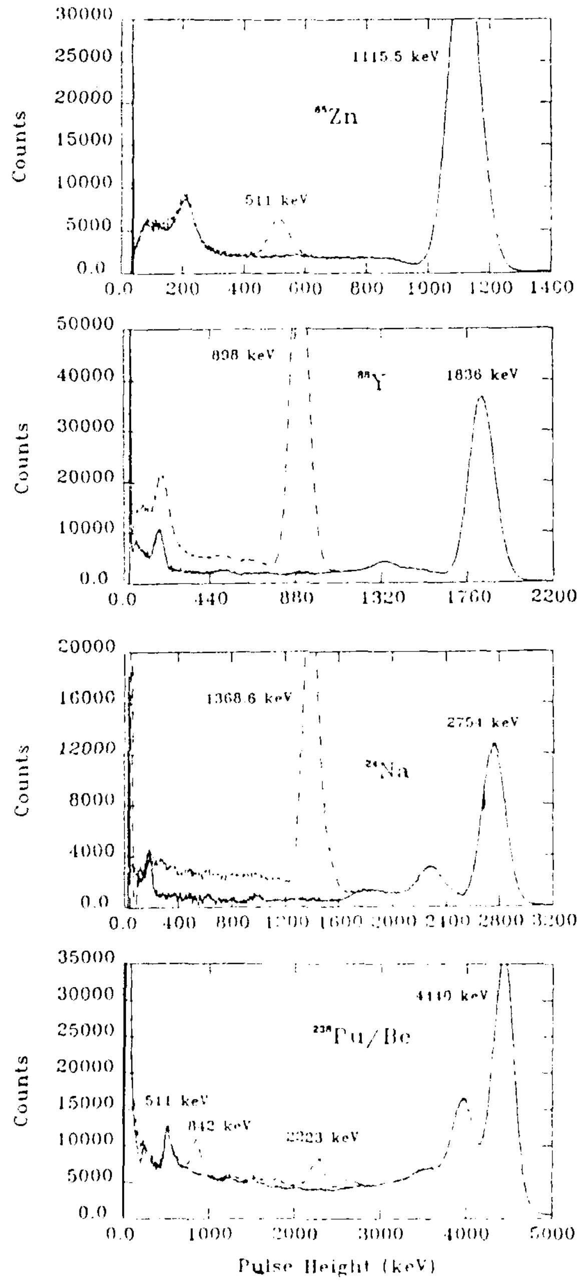

Figure 6. Pulse-height distributions acquired with a 7.62-cm x 7.62-cm BGO detector for gamma rays from ^{65}Zn, ^{88}Y, ^{24}Na, and $^{238}Pu/Be$ radioactive sources. The dashed curves include the effects of all gamma-rays emitted from the sources. The solid curves are the distributions from only the highest energy gamma rays emitted from the sources. The pulse-height distributions are not normalized.

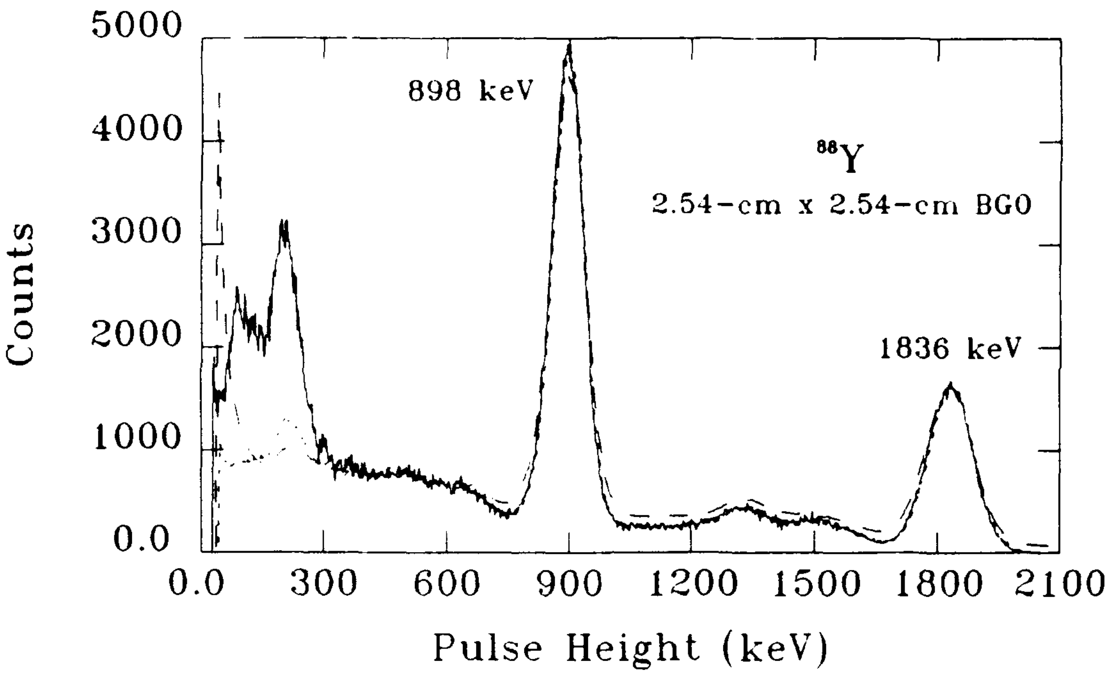

Figure 7. Pulse-height distributions acquired with a 2.54-cm x 2.54-cm BGO detector for gamma rays from a ^{88}Y radioactive source at various source-to-detector distances: 5 cm (dashed curve), 25 cm (dotted curve), and 100 cm (solid curve). The distributions are normalized relative to each other based on the 1836-keV photopeak.

involving only the calculation of the fractional solid angle subtended by a detector at the source and correcting for source decay. For the remaining sources, we normalized the single-energy pulse-height distributions for the 7.62-cm x 7.62-cm detector using the average intrinsic photopeak efficiencies for a 7.62-cm x 7.62-cm BGO detector with a source-to-detector distance of 91.7 cm reported in Ref. 17.

Table 4 compares the intrinsic photopeak efficiencies of Ref. 17 to those measured for our 7.62-cm x 7.62-cm detector. The photopeak areas needed to obtain the photopeak efficiencies in Table 4 were determined by fitting Gaussian functions to the data points above the half-maximum peak height values using MINUIT (Ref. 20). The agreement is good, especially at the higher gamma-ray energies and justifies the use of Ref. 17 for normalizing the distributions from the sources of unknown strength. The errors shown in the table result from a maximum 4.6 percent error in the known source intensities and a 2 percent error in photopeak areas.

Normalization of the pulse-height distributions from the uncalibrated sources for the 2.54-cm x 2.54-cm BGO detector using the efficiencies of Ref. 17 was accomplished as follows. We determined the photopeak areas of the uncalibrated source distributions for the 7.62-cm x 7.62-cm detector. Also, since these sources were extended sources, we performed Monte Carlo calculations to determine the various fractional solid angles subtended by the 7.62-cm x 7.62-cm detector at the sources. Using the fitted photopeak areas, computed solid angles, and the intrinsic efficiencies of Ref. 17, we could then determine the strength of the ^{24}Na, $^{238}Pu/Be$, $^{16}O^*$, and $^{12}C^*$ sources. We then used these strengths along with computed fractional solid angles and fitted photopeak areas pertaining to the 2.54-cm x 2.54-cm detector to normalize the single-energy pulse-height

Table 4. Instrinsic Photopeak Efficiencies At Various Gamma-Ray Energies For A 7.62-cm x 7.62-cm BGO Scintillation Detector.

Gamma-Ray Energy (keV)	Instrinsic Photopeak Efficiency Ref. 17 [(a)]	This Work [(b)]
123.6	0.75	0.89 ± 0.05
279.2	0.95	0.82 ± 0.04
514	0.90	0.85 ± 0.03
661.6	0.84	0.77 ± 0.03
834.8	0.79	0.81 ± 0.04
1115.5	0.71	0.71 ± 0.03
1274.5	0.67	0.64 ± 0.03
1836	0.57	0.61 ± 0.03

[(a)] Source-to-detector distance of 91.7 cm.
[(b)] Source-to-detector distance of 100 cm.

distributions from these sources for the 2.54-cm x 2.54-cm detector. The resultant intrinsic photopeak efficiencies for this detector are shown in Table 5. For the lowest gamma-ray energies through 1836 keV, the errors are a combination of a maximum 4.6 percent error in the known source intensities and a 2 percent error in photopeak areas. The remaining efficiencies include a 2 percent error in fractional solid angle, a 2 percent error in photopeak areas, and an estimated 5 percent error due to the normalization procedure. No intrinsic photopeak efficiency could be determined for the gamma-ray energy of 11.67 MeV because a photopeak was not observed due to the low efficiency at this energy and the resolution of the

Table 5. Instrinsic Photopeak Efficiencies At Various Gamma-Ray Energies For A 2.54-cm x 2.54-cm BGO Scintillation Detector With A Source-To-Detector Distance Of 100 cm.

Gamma-Ray Energy (keV)	Instrinsic Photopeak Efficiency
123.6	0.88 ± 0.05
279.2	0.82 ± 0.04
514	0.67 ± 0.02
661.6	0.52 ± 0.02
834.8	0.47 ± 0.02
1115.5	0.33 ± 0.01
1274.5	0.28 ± 0.01
1836	0.22 ± 0.01
2754	0.15 ± 0.01
4440	0.078 ± 0.004
6140	0.060 ± 0.003

detector. However, the 11.67-MeV pulse-height distribution is correctly normalized based on the photopeak area of the 4440-keV peak since the $^{12}C^{*}$ source emits 4440-keV and 11.67-MeV gamma rays of equal intensity.

IV. RESULTS AND DISCUSSION

Detector Energy Resolutions

An important parameter for the use of any scintillation detector is the percentage energy resolution as a function of gamma-ray energy. Figures 8 and 9, respectively, show the 2.54-cm x 2.54-cm and 7.62-cm x 7.62-cm BGO detector percentage resolutions as a function of gamma-ray energy. The resolutions were calculated for the full widths at half maximums of Gaussian-function fits to the photopeaks. The solid lines are least-squares fits to the data points and are given by

2.54-cm x 2.54-cm BGO detector:

$$\text{resolution (percent)} = 221.524 E_{\gamma}(\text{keV})^{-0.45135}$$

7.62-cm x 7.62-cm BGO detector:

$$\text{resolution (percent)} = 221.863 E_{\gamma}(\text{keV})^{-0.42863}.$$

A few of the measured resolution values for the 2.54-cm x 2.54-cm detector are 11.6 $\pm$ 0.2 percent, 7.2 $\pm$ 0.1 percent, and 4.5 $\pm$ 0.1 percent for gamma-ray energies of 661.6, 1836, and 6140 keV, respectively. The errors are 2 percent errors estimated from the variation in fits to the photopeaks. These resolution values agree well with the values reported in Ref. 13: 11.8 percent, 7.2 percent, and 4 percent. The corresponding measured resolution values for the 7.62-cm x 7.62-cm detector are 13.0 $\pm$ 0.3 percent, 8.5 $\pm$ 0.2 percent, and 5.6 $\pm$ 0.1 percent. Again, these values agree well with other reported values (Refs. 8,10,15,17), and in many cases

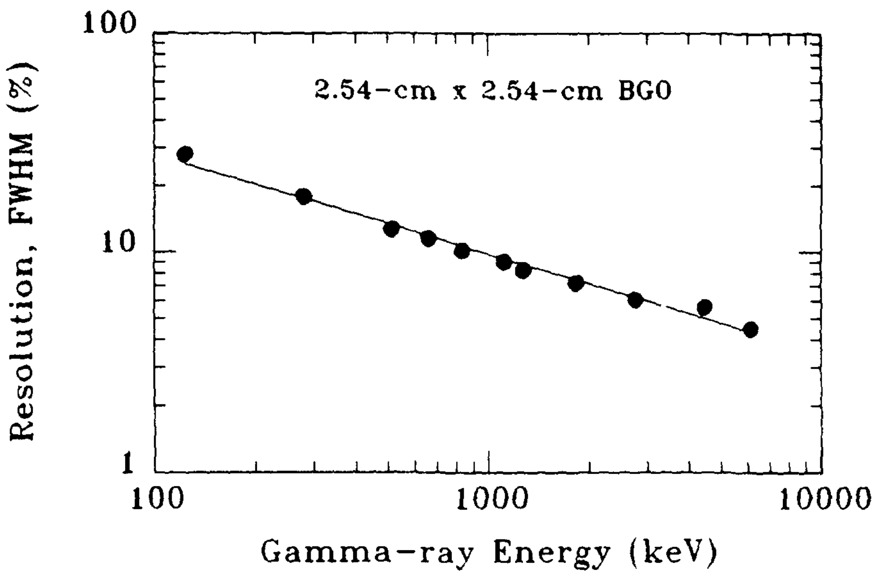

Figure 8. The percentage energy resolution (FWHM) as a function of gamma-ray energy for the 2.54-cm x 2.54-cm BGO scintillation detector.

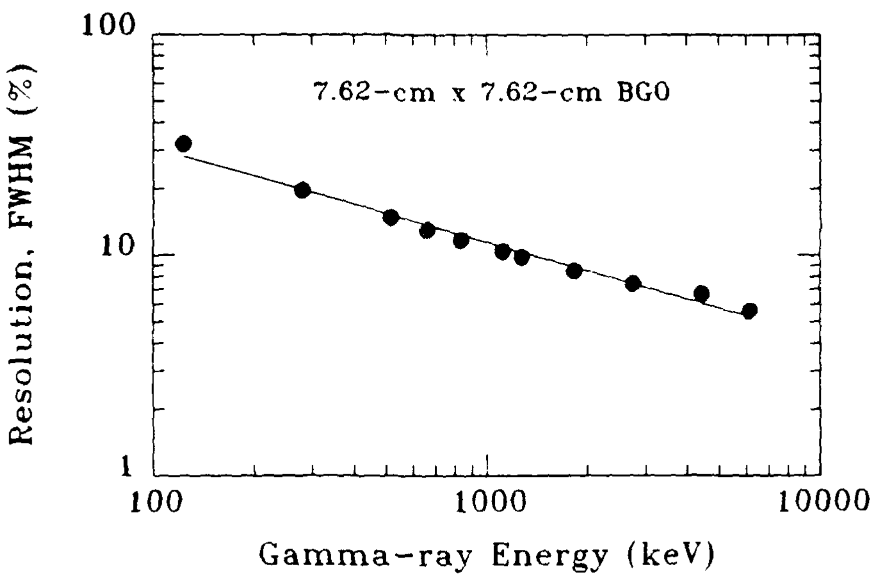

Figure 9. The percentage energy resolution (FWHM) as a function of gamma-ray energy for the 7.62-cm x 7.62-cm BGO scintillation detector.

our 7.62-cm x 7.62-cm BGO detector exhibits slightly better resolution.

Response Functions

The response function curves, plots of the number of counts recorded by the detectors per unit pulse height per incident gamma ray, are shown in Figs. 10 and 11 for the 2.54-cm x 2.54-cm BGO detector and Figs. 12 and 13 for the 7.62-cm x 7.62-cm BGO detector. Some general features are the following. The backscatter peak occurring at a pulse height of approximately 200 to 250 keV is a prominent feature of every response function for both detectors. The size of this peak in relation to the photopeak generally increases as the energy of the photopeak increases. Since the detectors were well removed from the walls, the ceiling, the floor, and any apparatus in the laboratory room during most of the response function measurements, the backscatter peaks must be primarily the result of Compton-scattered gamma rays from the detector housing materials and the aluminum camera tripods. Thus, the relative importance of the backscatter peak to a given response function will depend on the construction of the detector, its mounting, and other nearby materials and will obviously vary widely between detectors and experiments. A similar concern applies to the annihilation radiation occurring at a pulse height of 511 keV in all the response functions for $E_\gamma \geq$ 1836 keV. The annihilation peak arises from gamma-ray pair

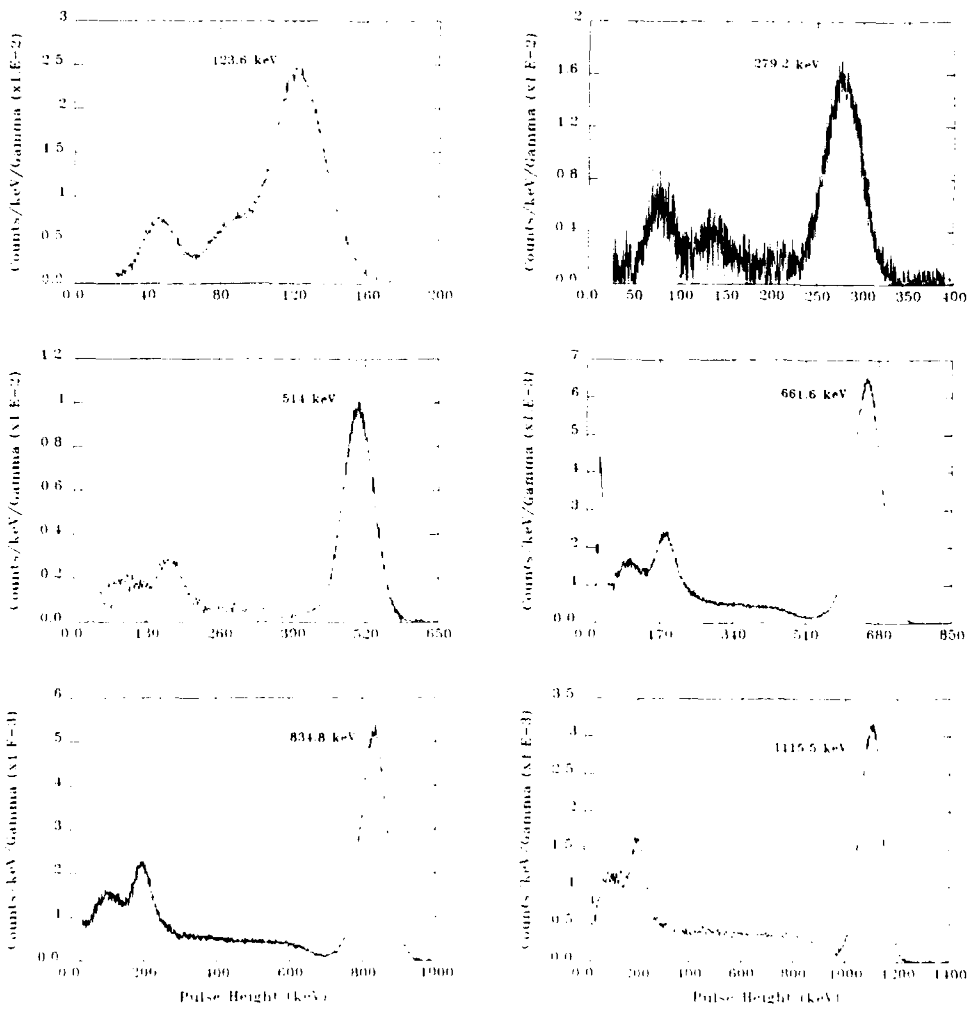

Figure 10. Response function curves for the 2.54-cm x 2.54-cm BGO scintillation detector at gamma-ray energies of 123.6, 279.2, 514, 661.6, 834.8, and 1115.5 keV.

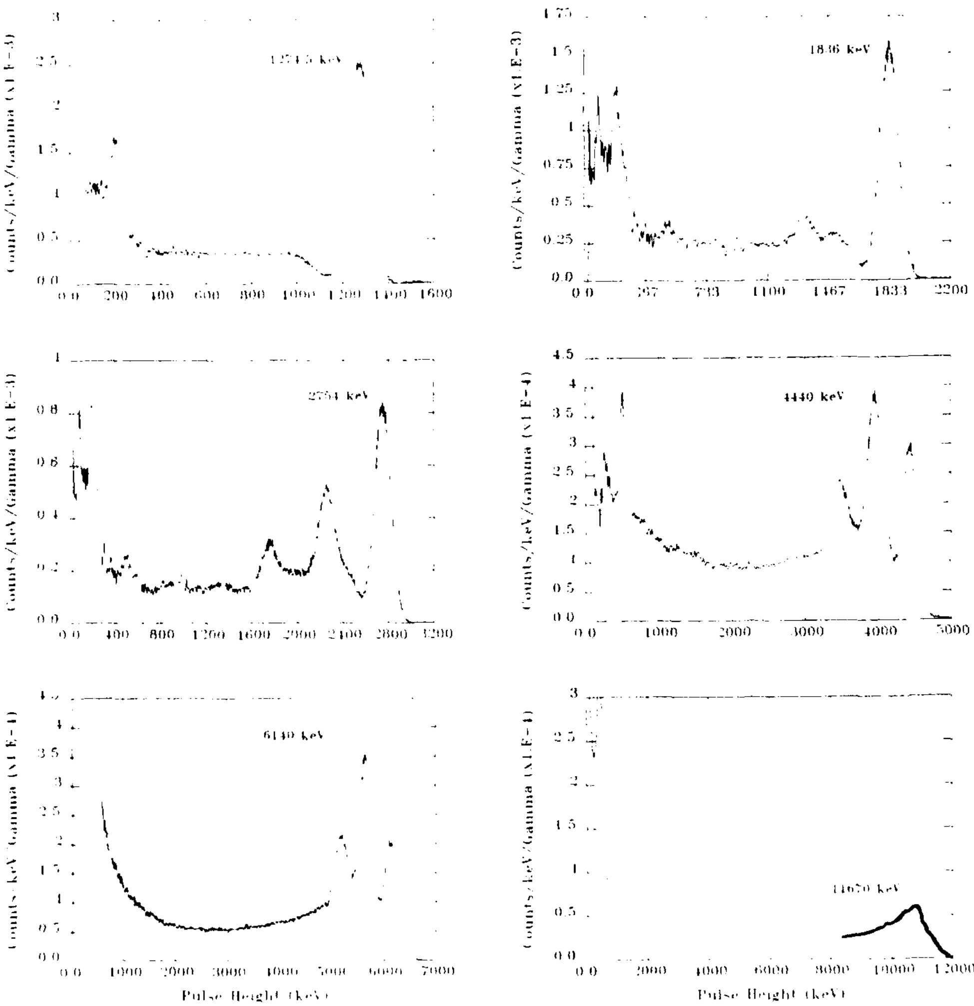

Figure 11. Response function curves for the 2.54-cm x 2.54-cm BGO scintillation detector at gamma-ray energies of 1274.5, 1836, 2754, 4440, 6140, and 11,670 keV.

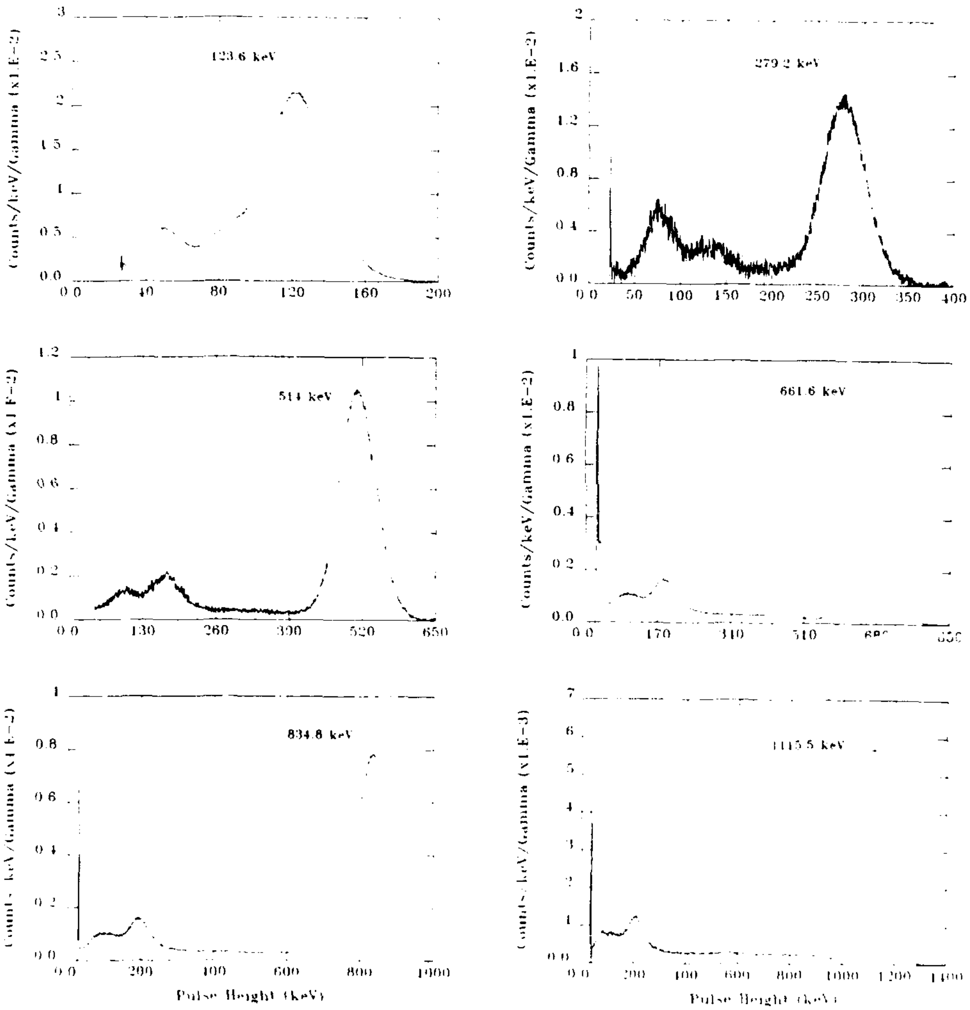

Figure 12. Response function curves for the 7.62-cm x 7.62-cm BGO scintillation detector at gamma-ray energies of 123.6, 279.2, 514, 661.6, 834.8, and 1115.5 keV.

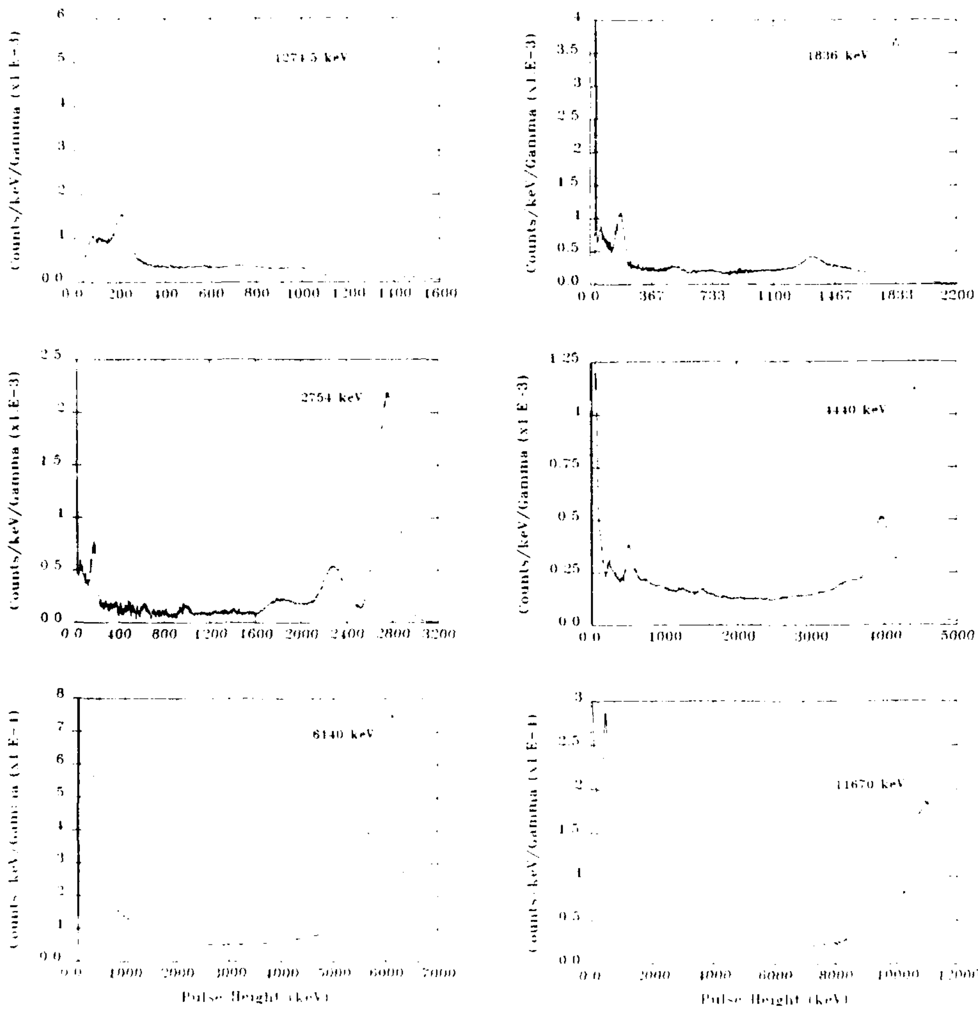

Figure 13. Response function curves for the 7.62-cm x 7.62-cm BGO scintillation detector at gamma-ray energies of 1274.5, 1836, 2754, 4440, 6140, and 11,670 keV.

production within materials surrounding the detector and again is specific to the detector and experiment. Therefore, the pulse-height regions below approximately 600 keV of the response functions shown in Figs. 10-13 are detector specific. When using these response functions to unfold gamma-ray spectra, the unfolded results below 600 keV should be used with caution.

For the 2.54-cm x 2.54-cm BGO detector, the single- and double-escape peaks become prominent at E_γ = 2754 keV and dominate the response at $E_\gamma \geq$ 4440 keV. However, the photopeak is always the dominant part of the response function for the 7.62-cm x 7.62-cm BGO detector. Also, the Compton regions of the 7.62-cm x 7.62-cm responses are significantly less important than these regions for the 2.54-cm x 2.54-cm detector. Therefore, if energy resolution is not a primary concern, the larger BGO detector is a better choice for measurement of continuous gamma-ray spectra since most of its response is in the photopeak. A photopeak-dominated response can be used to simplify the unfolding process and, at the least, results in better unfolded results using response matrix unfolding methods. However, the energy resolution of the 2.54-cm x 2.54-cm detector is better than the 7.62-cm x 7.62-cm detector and should be used when resolution is important. Also, its smaller size makes it a better choice when measuring high-energy gamma rays accompanied by large numbers of neutrons and low-energy gamma rays.

V. RESPONSE MATRIX GENERATION

Technique

In order to use the response functions reported in this paper for unfolding measured gamma-ray spectra, most unfolding methods require a response matrix. The general response matrix is a square matrix of elements with each element denoted by $R_{jk}(P_j,E_k)$. $R_{jk}(P_j,E_k)$ is the probability that a gamma ray incident on the detector with an energy in the bin E_k of width ΔE_k produces a count in the pulse-height distribution in the bin P_j of with ΔP_j. $R_{jk}(P_j,E_k)$ is related to the measured gamma-ray spectrum and incident gamma-ray spectrum through the relation

$$M_j(P_j) = \sum_k R_{jk}(P_j,E_k)T_k(E_k),$$

where the $T_k(E_k)$'s compose a discrete representation of the gamma-ray spectrum incident on the detector and the $M_j(P_j)$'s are a discrete representation of the measured spectrum. To determine the incident gamma-ray spectrum, T(E), the response elements, $R_{jk}(P_j,E_k)$, are needed at many energy and pulse-height values for arbitrary ΔP_j and ΔE_k bin widths. We measured the response functions for 2.54-cm x 2.54-cm and 7.62-cm x 7.62-cm BGO detectors at only 12 energy values spanning the gamma-ray energy range of 123.6 keV to 11.67 MeV. However, using an interpolation procedure discussed below, these 12 responses can be used to obtain response functions at any values of gamma-ray energy between 123.6

keV and 11.67 MeV and thus for construction of a response matrix.

Generation of a response function for a gamma-ray energy not measured is done as follows. Suppose that one column of the response matrix is the energy bin centered at E_k = 5290 keV. This energy is bracketed by the energies of 4440 keV and 6140 keV for which measured response functions exist. The 5290-keV response function is interpolated from the 4440-keV and 6140-keV response functions. First, the 4440-keV response function curve is shifted and stretched along the pulse-height axis so that the positions of the photopeak (4440 keV), single-escape peak (3929 keV), and double-escape peak (3418) acquire new positions corresponding to the positions of these three peaks for a 5290-keV response function--5290 keV, 4779 keV, and 4268 keV, respectively. The magnitude of the 4440-keV response curve is not modified. Also, the positions of the backscatter and annihilation peaks in the shifted and stretched 4440-keV response curve are left unchanged. The above is accomplished by dividing the pulse-height axis of the 4440-keV response curve into three regions: pulse heights $\leq$ 733 keV, pulse heights $\geq$ 3167 keV, and 733 < pulse heights < 3167 keV. These regions are for the 2.54-cm x 2.54-cm BGO response function and are different for the 7.62-cm x 7.62-cm detector because of the difference in energy resolution. The transformation from the 4440-keV response to the shifted and stretched 4440-keV response

consists of leaving the pulse heights $\leq$ 733 keV untouched, adding a constant value of 850 keV (5290 - 4440) to all pulse heights $\geq$ 3167 keV, and multiplying the middle region of pulse heights by the appropriate stretching factor to fill the middle region of the shifted and stretched 4440-keV response pulse-height axis. This procedure maintains the shapes of all peaks in the response function. Second, the 6140-keV response function curve is shifted and compressed in the manner described above except that the shift is negative and the three pulse-height regions are changed so as to correspond to the shape of the 6140-keV response. After this shifting and stretching or compressing procedure, the 4440-keV and 6140-keV pulse-height axes appear as 5290-keV pulse-height axes. The final step is a weighted logarithmic interpolation of the ordinate axes of the modified 4440-keV and 6140-keV response curves on a point-by-point basis along the common pulse-height axis to determine the 5290-keV curve. The weighting is based on the natural logarithm of the energy differences between the three response functions. The interpolation procedure just described is shown graphically in Fig. 14 for the 2.54-cm x 2.54-cm BGO detector.

To test this construction procedure for response functions, response functions for both detectors were generated at gamma-ray energies for which measured response functions existed. Two examples for the 2.54-cm x 2.54-cm BGO detector at E_γ = 1274.5 and 4440 keV are shown in Figs.

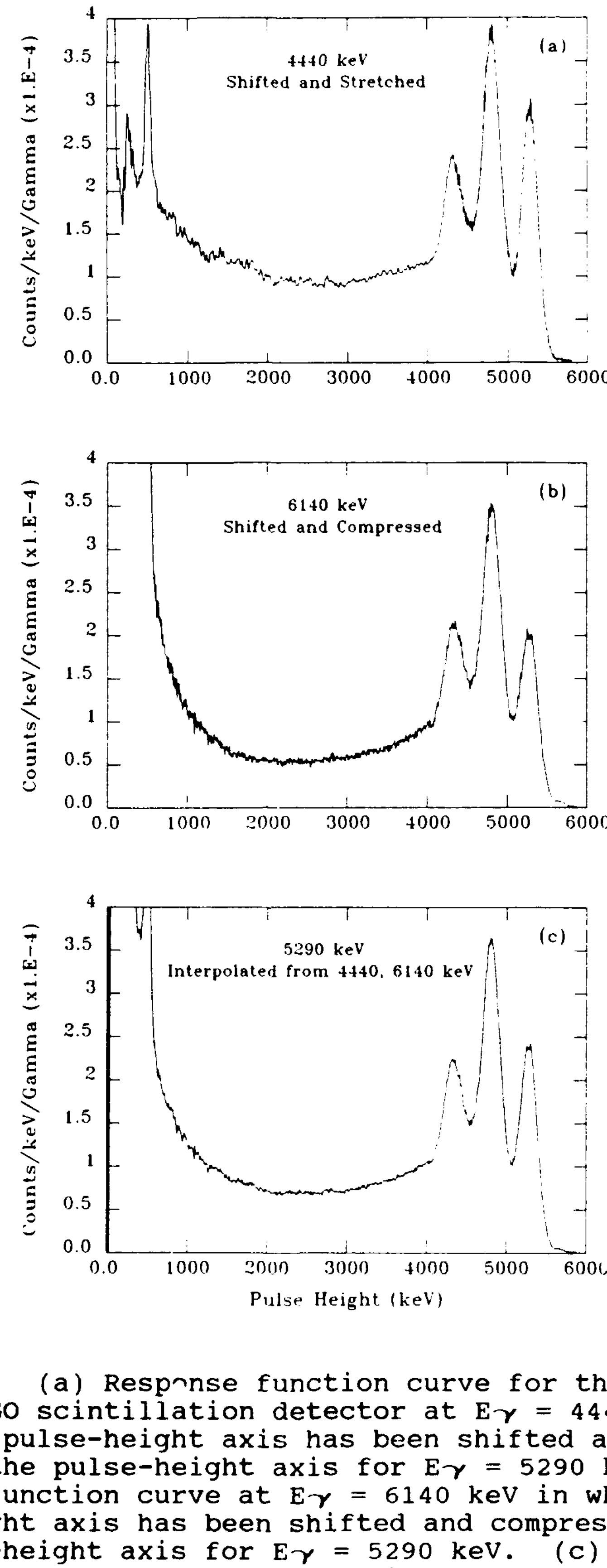

Figure 14. (a) Resp^nse function curve for the 2.54-cm x 2.54-cm BGO scintillation detector at E_γ = 4440 keV in which the pulse-height axis has been shifted and stretched to match the pulse-height axis for E_γ = 5290 keV. (b) Response function curve at E_γ = 6140 keV in which the pulse-height axis has been shifted and compressed to match the pulse-height axis for E_γ = 5290 keV. (c) Response function curve at E_γ = 5290 keV interpolated from the 4440- and 6140-keV response curves.

15 and 16. The interpolated 1274.5-keV response used the measured 1115.5-keV and 1836-keV response functions; the interpolated 4440-keV response used the measured 2754-keV and 6140-keV response functions. There is very good agreement between the interpolated and measured response curves, especially for pulse heights greater than 600 keV where the responses are applicable.

The interpolated response functions are the basis for obtaining the response matrix. A column of the matrix $R_{jk}(P_j,E_k)$ is the appropriately binned (in pulse height) response function for a gamma-ray energy of E_k and energy bin width of ΔE_k. We chose to construct this response function as a simple average of three interpolated response functions at energy values equally spaced within ΔE_k centered at E_k. The averaging process accounts for possible variations in the detector response function across the energy bin ΔE_k. These variations are most important if ΔE_k is large. Also, the averaging process effectively smooths the response function, which is beneficial in the unfolding process.

Computer Code

The computer code, BGRESP, used to construct the response matrix for a 2.54-cm x 2.54-cm or 7.62-cm x 7.62-cm BGO detector using the experimental response functions reported in this paper is listed in the Appendix. The code is written in FORTRAN-77. The primary components of the

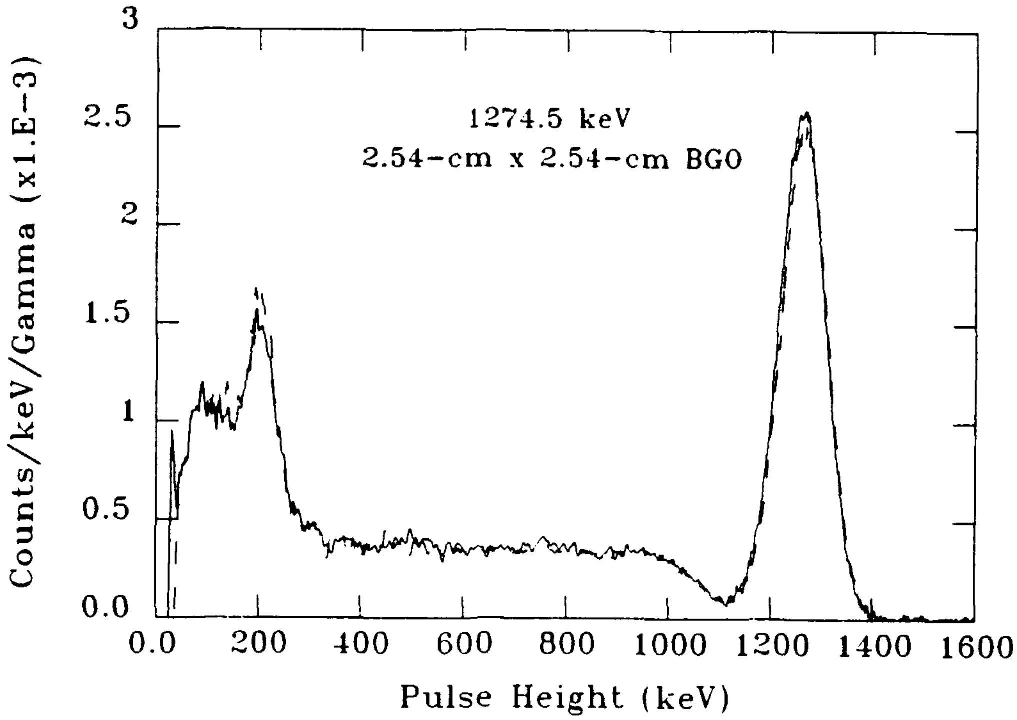

Figure 15. Measured (dashed curve) and interpolated (solid curve) response functions for the 2.54-cm x 2.54-cm BGO scintillation detector at E_γ = 1274.5 keV.

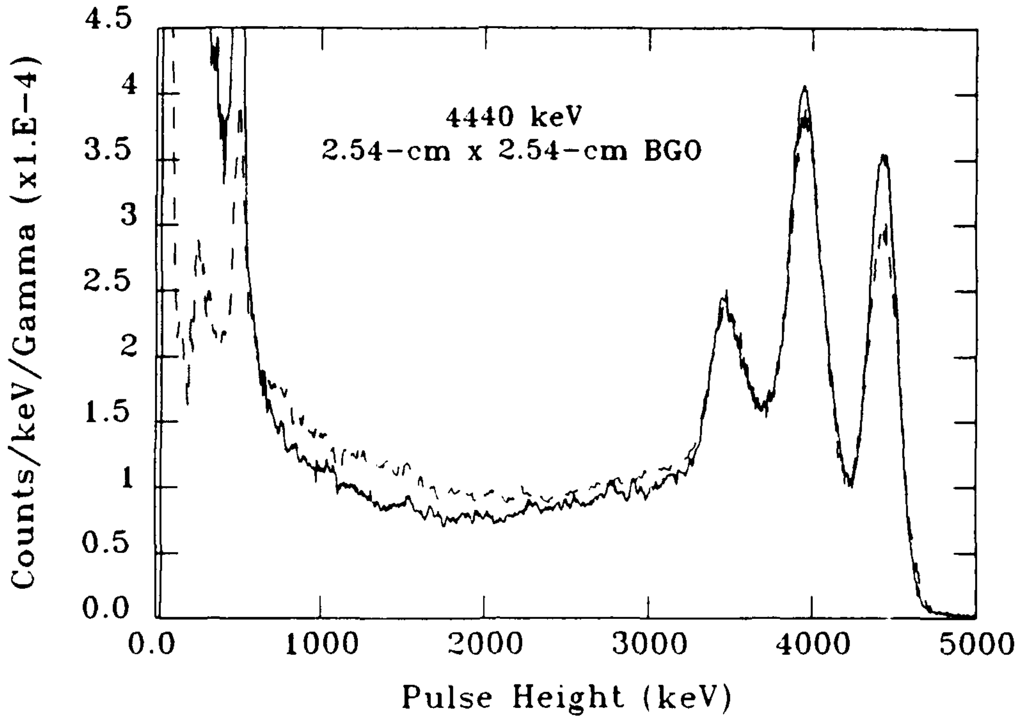

Figure 16. Measured (dashed curve) and interpolated (solid curve) response functions for the 2.54-cm x 2.54-cm BGO scintillation detector at E_γ = 4440 keV.

code are the main program and two subroutines. The two subroutines are identical except for the initial data statements. One subroutine constructs an interpolated response function for the 2.54-cm x 2.54-cm detector at a specific value of gamma-ray energy passed by the main program; the other subroutine constructs interpolated response functions for the 7.62-cm x 7.62-cm detector. The main program forms the response matrix from the interpolated response functions. The maximum size allowed for the response matrix is 100 gamma-ray energy bins by 100 pulse-height bins.

The input and output of the code is simple. The input consists of specifying one of the two sizes of BGO detectors, 2.54-cm x 2.54-cm or 7.62-cm x 7.62-cm, the range of pulse heights for generation of the response matrix, and the energy and pulse-height bin width of a response matrix element. This bin width is entered as a multiple of the detector energy resolution. Thus the response matrix contains variable bin widths which increase as the energy increases. The output consists of three files: (1) a listing of the response matrix in an easily readable format, (2) a listing of the response matrix in a compact format for input to an unfolding code, and (3) a listing of the bin values of the response matrix for input to an unfolding code.

Several tests of the response matrix computer code were performed for both BGO detectors. Fig. 17 is an

illustration of one such test. Fig. 17a is a simulated pulse-height distribution from the 2.54-cm x 2.54-cm detector due to six equal-intensity gamma-ray sources obtained by adding together the measured response functions at E_γ = 834.8, 1274.5, 1836, 2754, 4440, and 6140 keV. These six response functions were scaled before adding them together so that each one corresponds to the same number of gamma rays incident on the detector. Furthermore, the resultant pulse-height distribution was normalized to correspond to a total of six gamma rays incident on the detector. The fact that the simulated pulse-height distribution is a sum of measured response functions instead of an actual simultaneous measurement of six gamma-ray sources is not a limitation of the test since the response matrix results from an averaging of the interpolated response functions and not directly from the measured response functions. Unfolding this pulse-height distribution with the 2.54-cm x 2.54-cm response matrix should ideally yield a gamma-ray spectrum, T(E), consisting of six delta-function peaks at the energies 834.8, 1274.5, 1836, 2754, 4440, and 6140 keV, each peak with an area of one. Fig. 17b shows the unfolded spectrum T(E). A response matrix of size 92 x 92 with a bin width of 0.5 times the detector resolution, generated from the code BGRESP, was used for the unfolding. The areas of the peaks are 1.03, 0.96, 0.89, 1.00, 1.02, and 1.03, for a total area of 5.93 as compared to the ideal expected total area of 6. A test

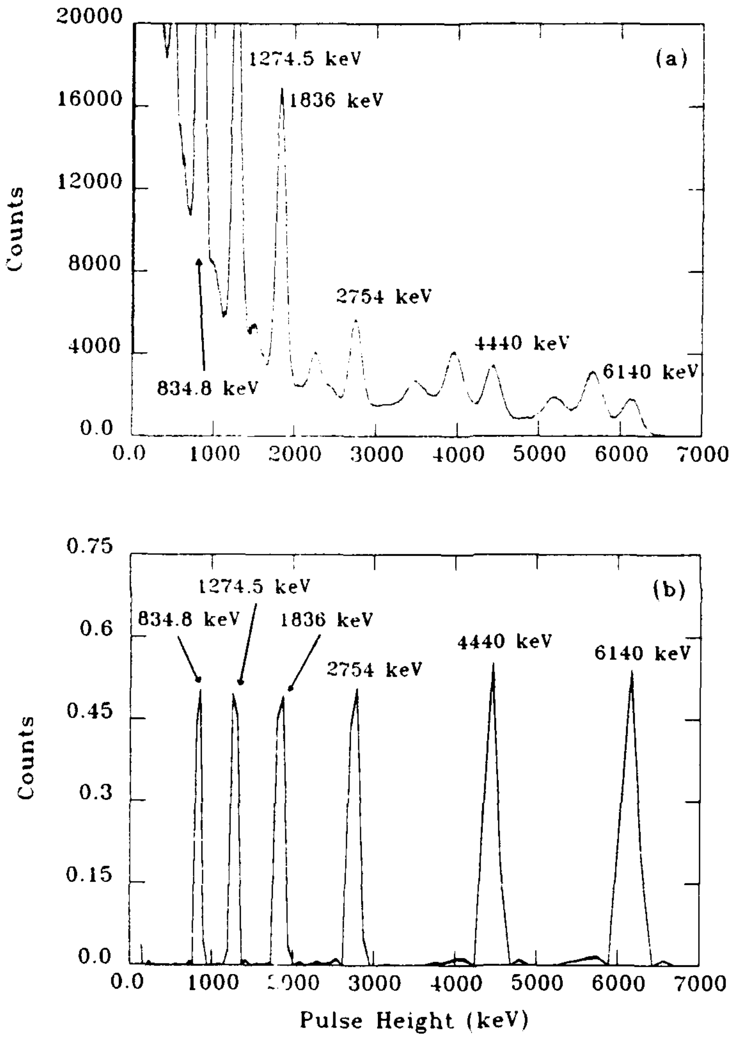

Figure 17. (a) A simulated, measured pulse-height distribution resulting from equal-intensity gamma-ray sources at E_γ = 834.8, 1274.5, 1836, 2754, 4440, and 6140 keV for the 2.54-cm x 2.54-cm BGO scintillation detector. (b) The unfolded gamma-ray spectrum resulting from (a) using a 92 x 92 response matrix constructed from measured 2.54-cm x 2.54-cm response functions.

of the response matrix computer code for the 7.62-cm x 7.62-cm BGO detector is illustrated in Fig. 18. Again, Fig. 18a is a simulated pulse-height distribution from the 7.62-cm x 7.62-cm detector due to six equal-intensity gamma-ray sources. Fig. 18b shows the unfolded spectrum. A response matrix of size 77 x 77 with a bin width of 0.5 times the detector resolution was used for the unfolding. The areas of the peaks are 0.996, 1.02, 1.04, 1.04, 1.00, 1.04, for a total area of 6.14 as compared to the ideal expected total area of 6.

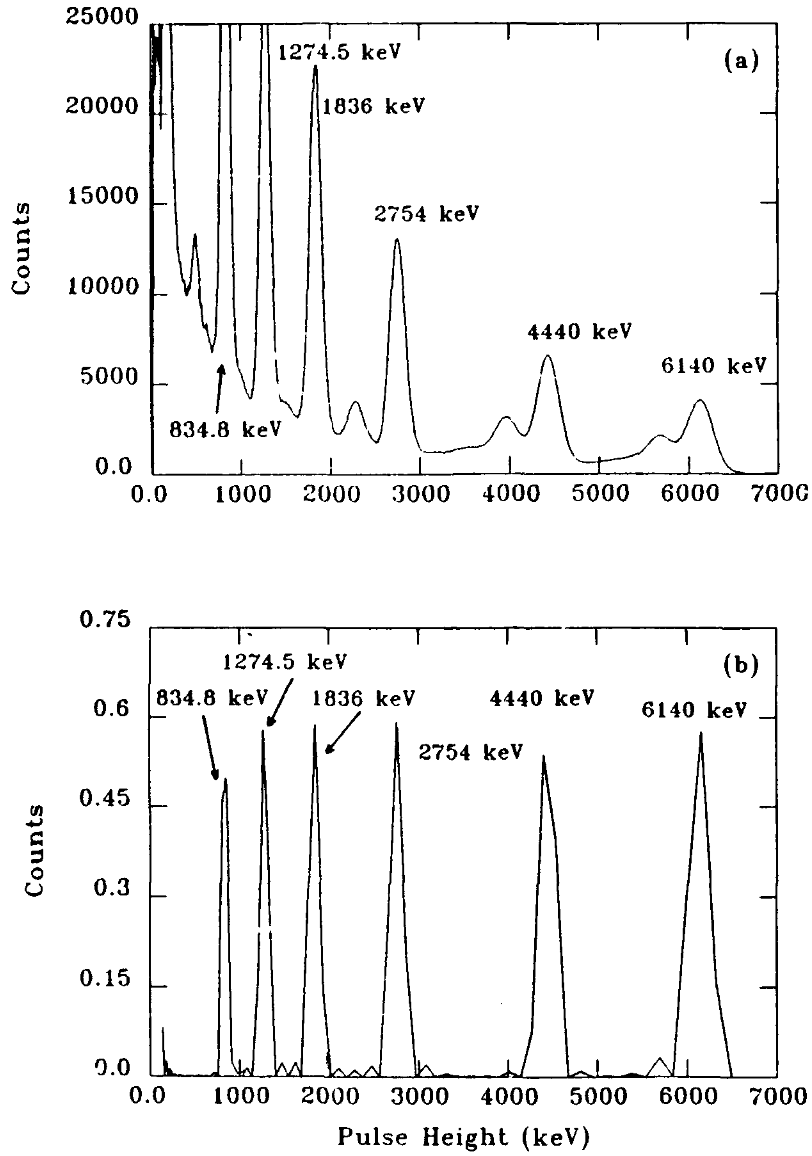

Figure 18. (a) A simulated, measured pulse-height distribution resulting from equal-intensity gamma-ray sources at E_γ = 834.8, 1274.5, 1836, 2754, 4440, and 6140 keV for the 7.62-cm x 7.62-cm BGO scintillation detector. (b) The unfolded gamma-ray spectrum resulting from (a) using a 77 x 77 response matrix constructed from measured 7.62-cm x 7.62-cm response functions.

VI. CONCLUSIONS

The 24 response functions spanning the gamma-ray energy range of 123.6 keV to 11.67 MeV for 2.54-cm x 2.54-cm and 7.62-cm x 7.62-cm BGO scintillation detectors along with the computer code for calculating response matrices should be useful by others possessing the same size detectors with equivalent or worse detector resolutions. They can be used for unfolding gamma-ray pulse-height distributions with reliable results for pulse heights above 600 keV. Since the response functions correspond to a source-to-detector distance of 100 cm, they can be used with little error for any experimental arrangement in which the BGO detector is at least approximately 50 cm from the source.

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APPENDIX

The following computer code, BGRESP, calculates a response matrix for a 2.54-cm x 2.54-cm or 7.62-cm x 7.62-cm BGO detector using a set of 12 experimental response functions for each detector covering the gamma-ray energy range of 123.6 keV to 11.67 MeV. A copy of this code along with the 24 response function data files and sample input and output can be obtained from the authors by writing to their address or calling the number shown in the listing of the code.

```
      PROGRAM BGRESP
C
C by:
C     Rex R. Kiziah
C     Physics Department
C     United States Air Force Academy
C     Colorado Springs, CO 80840-5701
C     (719) 472-3355
C
C VERSION 1.0,  3 April 1990
C
C Program BGRESP is a FORTRAN code to calculate a gamma-ray response
C matrix for a 2.54-cm x 2.54-cm or a 7.62-cm x 7.62-cm BGO inorganic
C scintillator. The maximum matrix size is 100 x 100 and can span any
C energy and pulse-height range between 123.6 and 11670.0 keV.  The
C matrix is in a format such that the columns are pulse-height bins and
C the rows are energy bins (each in keV). The 2-D array FINRESP(I,J)
C contains the matrix (I=1,100,J=1,100 max).  FINRESP(I',J') is the
C probability for one gamma ray in the energy bin I' to produce a count
C in the pulse-height bin J'.  The matrix is calculated via interpolation
C from 12 data files for each detector.  Each data file is an experimental
C response function--background corrected, pulse height calibrated,
C properly normalized, etc.  The response functions were acquired for a
C 2.54-cm x 2.54-cm and a 7.62-cm x 7.62-cm BGO scintillator, each
C purchased from BICRON Corporation.  The resolution for the 2.54-cm x
C 2.54-cm detector was determined experimentally to be R = DeltaE/E =
C 2.21524*E(gamma)**-0.45135, with E(gamma) in keV.  For the 7.62-cm x
C 7.62-cm detector, R = 2.21863*E(gamma)**-0.42863.  If this code is to
C be used for a BGO detector with poorer resolution, the user should first
C fold the data files with a Gaussian of the proper width to simulate
```

```
C the poorer resolution effects and then appropriately modify the
C resolution formulas used in this code.
C
C Running the Code:
C
C (1) The user must have the 24 response function data files, 12 for
C     the 2.54-cm x 2.54-cm BGO detector and 12 for the 7.62-cm x
C     7.62-cm detector.
C (2) If the user's detector resolution is worse than that given above
C     in the description of the code, appropriate modifications to the
C     data files and this code must be made.
C (3) Run BGRESP.  The user is prompted for all necessary information.
C (4) Three output files are generated.  BGRESP.OUT--a file listing the
C     response matrix bins and the response matrix in a format for ease
C     of reading.  BGO.MAT--a file containing the response matrix
C     in a format for input to an unfolding code.  BGLIMS.DAT-- a file
C     containing the number of bins and the response matrix bin
C     values (in MeV) for input to an unfolding code.
C
C Program Summary:
C
C Subroutines Used:
C
C     BG1RESP  == Calculates a gamma-ray response function for any
C                 gamma-ray energy in the range 123.6 to 11670.0 keV
C                 for the 2.54-cm x 2.54-cm BGO detector.  The 1-D array
C                 RESP(I), I=1000, contains the response function.
C                 RESP(I') is the probability per unit pulse height (in
C                 keV) that one gamma ray of the specified energy
C                 produces a count in the pulse height bin I'.  That is,
C                 RESP(I') has units of Counts/keV/gamma-ray.
C     BG3RESP  == Identical to BG1RESP except that the calculation is for
C                 the 7.62-cm x 7.62-cm BGO detector.
C
C Functions Used:
C
C     TERP     == An interpolation routine.
C
C Common Blocks Used:
C
C     DATARAYS == Contains 2 1-D arrays shared between the main program
C                 and the two subroutines.
C     PARM     == Contains 3 parameters shared between the main program
C                 BGRESP and the subroutines BG1RESP and BG3RESP.
C
C Local Variables Used:
C
C     BINC     == Real variable.  A variable for storing the centroid
C                 of each bin of the response matrix.  In keV.
C     BINFAC   == Real variable.  Factor for determining individual
C                 bin sizes for response matrix.  It is a multiple of
C                 the detector energy resolution--thus, bin sizes are
C                 unequal.
C     BINW     == Real variable.  A variable for storing the width
```

```
C                    of each bin of the response matrix.  In keV.
C     DELTA    == Real variable.  Used in computation of energy values
C                    for the response functions used in construction of
C                    the response matrix.
C     ERESP    == Real 1-D 300 element array.  Contains the energy values
C                    (in keV) for generation of response functions used to
C                    construct the response matrix.
C     FINRESP  == Real 2-D 100 x 100 array.  Contains the response matrix.
C                    FINRESP(I',J') is the probability that a gamma ray in
C                    the energy bin I' produces a count in the pulse-height
C                    bin J'.  Both bin widths are in keV.
C     IDET     == Integer variable.  Value is 1 or 2.  1 is for 2.54-cm
C                    x 2.54-cm detector; 2 is for 7.62-cm x 7.62-cm.
C                    Determines for which detector the response matrix is
C                    computed.
C     MM       == Integer variable.  A counter.
C     NBINS    == Integer variable.  The # of bins in the response matrix.
C                    Cannot exceed 100.
C     NN       == Integer variable.  A counter.
C     NRESP    == Integer variable.  This is the number of response
C                    functions used to compute the response matrix.
C                    NRESP=NBINS*3.  3 response functions per bin are used.
C     PHBIN    == Real variable.  In Common Block PARM.  The pulse-height
C                    bin width (in keV) for each response function computed
C                    in BG1RESP or BG3RESP.
C     PHLIMS   == Real 1-D 101 element array.  Contains the pulse-
C                    height values (in keV) for the response matrix bins.
C     PRANH    == Real variable.  In Common Block PARM.  Highest value
C                    of pulse height (in keV) for generation of response
C                    matrix.
C     PRANL    == Real variable.  In Common Block PARM.  Lowest value
C                    of pulse height (in keV) for generation of response
C                    matrix.
C     RBRESP   == Real 1-D 100 element array.  Same as RESP with
C                    the bin width integrated out, but rebinned to the
C                    response matrix bin sizes.
C     RESP     == Real 1-D 1000 element array.  In Common Block
C                    DATARAYS.   Array to store response function for a
C                    given gamma-ray energy.  RESP(I') is the
C                    probability per unit pulse height (in keV) for one
C                    gamma ray to produce a count in the pulse height bin
C                    I'.  Also stores the same response function with the
C                    bin width integrated out--yields probability instead
C                    of probability per unit pulse height.
C     RESPPH   == Real 1-D 1000 element array.  In Common Block
C                    DATARAYS.  Contains the pulse-height bin values
C                    (in keV) for each response function computed by
C                    BG1RESP or BG3RESP.
C     SUBRESP  == Real 2-D 300 x 100 array.  Contains all the response
C                    functions, RBRESP, needed for computation of the
C                    response matrix.  300--max # of response functions.
C     SUMHI    == Real variable.  Used for the rebinning of response
C                    function computed by BG1RESP or BG3RESP to bins
C                    of the response matrix.
```

```
C      SUMLO    == Real variable.  Used for the rebinning of response
C                  function--see SUMHI.
C      SUMRESP  == Real 1-D 1000 element array.  A running sum of RESP.
C                  SUMRESP(I)=RESP(I)+RESP(I-1)+RESP(I-2)+....  Used
C                  for rebinning of response function--see SUMHI.
C      TYPE     == Character variable.  Either '2.54-cm x 2.54-cm' or
C                  '7.62-cm x 7.62-cm'.
C
C  Set up the common blocks used between the main program and the
C  subroutines BG1RESP and BG3RESP.
C
      COMMON/PARM/PRANL,PRANH,PHBIN
      COMMON/DATARAYS/RESP(1000),RESPPH(1000)
C
C  Dimension the variables.
C
      CHARACTER TYPE*17
      DIMENSION PHLIMS(101),ERESP(300),SUMRESP(1000),RBRESP(100),
     1          SUBRESP(300,100),FINRESP(100,100)
C
C  Establish the unit for interactive input of data needed by the code.
C
      CALL ASSIGN(1,'SYS$OUTPUT')
C
C  Prompt user for required input:  Detector, Response Matrix Size,
C  Response Matrix Bin Size
C
5     WRITE(1,10)
10    FORMAT(1X,'Enter the detector for which response matrix is'
     1 ' to be generated:'/1X,'(1) for 1" x 1", (2)'
     2 ' for 3" x 3":  ',$)
      READ(1,*) IDET
      IF(IDET.NE.1.AND.IDET.NE.2) GO TO 5
25    WRITE(1,30)
30    FORMAT(1X,'Enter the range of pulse heights (in keV) for',
     1 ' generation of response matrix---'/1X,'Must be between',
     2 ' 123.6 and 11670.0 keV:  ',$)
      READ(1,*) PRANL,PRANH
      IF(PRANL.LT.123.6.OR.PRANH.GT.11670.0.OR.PRANL.GE.PRANH)
     1 GO TO 25
45    WRITE(1,50)
50    FORMAT(1X,'Enter factor for determination of response',
     1 ' matrix bin size as a'/1X,'multiple (< or > 1) of the',
     2 ' detector energy resolution:  ',$)
      READ(1,*) BINFAC
C
C  Establish for which detector the response matrix is being computed
C  and set up the character variable TYPE for output identification
C  purposes.
C
      IF(IDET.EQ.1) THEN
          TYPE = '2.54-cm x 2.54-cm'
      ELSE
          TYPE = '7.62-cm x 7.62-cm'
```

```
      ENDIF
C
C  Establish the pulse-height values (in keV) of the response matrix
C  bins.  The bin sizes are based on the detector resolution--they
C  are a user-specified multiple, BINFAC, of the detector resolution.
C  Also, count the number of bins that are established and store
C  this number in NBINS.
C
      NBINS = 0
      PHLIMS(1) = PRANL
      DO 70 I = 2,101
          NBINS = NBINS + 1
          IF(IDET.EQ.1) THEN
              PHLIMS(I) = BINFAC*2.21524*PHLIMS(I-1)**0.54865
     1                  + PHLIMS(I-1)
          ELSE
              PHLIMS(I) = BINFAC*2.21863*PHLIMS(I-1)**0.57137
     1                  + PHLIMS(I-1)
          ENDIF
          IF(PHLIMS(I).GT.PRANH) THEN
              PHLIMS(I) = PRANH
              GO TO 75
          ENDIF
70    CONTINUE
75    CONTINUE
C
C  Check to make sure that the input provided by the user for the
C  pulse-height range of the response matrix and the bin size are
C  compatible--that is, the max size of the response matrix is
C  100 x 100 and too small of a bin size will result in a response
C  matrix of size 100 x 100, yet it will not span the entire range
C  of pulse heights.
C
      IF(PHLIMS(NBINS+1).LT.PRANH) THEN
          WRITE(1,80)
80        FORMAT(1X,'Factor for determination of response matrix',
     1    ' resolution is too small!'/1X,'RESPONSE MATRIX SIZE',
     2    ' EXCEEDS THE ALLOWED SIZE')
          GO TO 45
      ENDIF
C
C  The energy and pulse-height bins of the response matrix have now
C  been established.  For each energy bin, a pulse-height integrated
C  response function is needed.  Since the energy bins may be wide
C  in some cases, to use as much of the experimental data as possible,
C  the code uses three pulse-height integrated response functions for
C  each energy bin.  The procedure is as follows:  Divide each energy
C  bin into 6 equal sub-bins.  Then compute a pulse-height integrated
C  response function at the energy value of the first, third, and fifth
C  sub-bins.  Average these three response functions to obtain the
C  response function for one energy bin of the response matrix.  The
C  following loop sets up the energy values for computation of the
C  3*NBINS response functions.
C
```

```
      NRESP = 3*NBINS
      DO 100 I = 2,NBINS+1
           DELTA = PHLIMS(I) - PHLIMS(I-1)
           DELTA = DELTA/6.0
           DO 90 J = 1,3
                K = I-2
                ERESP(3*K+J) = PHLIMS(I-1) + (2*J - 1)*DELTA
90         CONTINUE
100   CONTINUE
C
C  This loop calls either the subroutine BG1RESP or BG3RESP to compute
C  the 3*NBINS pulse-height integrated response functions.  The energy
C  value is the only variable passed directly to the subroutines.  The
C  other variables needed are passed through the common blocks PARM and
C  DATARAYS.  Each subroutine returns a response function in the array
C  RESP.  RESP is an absolute differential efficiency curve for the
C  appropriate detector--that is, a curve of counts per unit pulse height
C  (in keV) per incident gamma ray.  RESP is returned via the common
C  block DATARAYS.  Also returned is the array RESPPH which contains
C  the 1000 pulse height values at which the response function is computed.
C  Lastly, the constant bin width of the response function, PHBIN is
C  returned via the common block PARM.  This bin width is needed so that
C  the absolute differential efficiency curve can be converted to an
C  absolute efficiency curve, i.e., a pulse-height integrated response
C  function.
C
      DO 150 I = 1,NRESP
           IF(IDET.EQ.1) THEN
                CALL BG1RESP(ERESP(I))
           ELSE
                CALL BG3RESP(ERESP(I))
           ENDIF
C
C  The response function has been computed.  Now, integrate over pulse
C  height and store the pulse-height integrated response back into the
C  array RESP.  RESP(I') is now the probability that a gamma ray of
C  the specified energy for which RESP was computed produces a count
C  in the pulse height bin I'.
C
           DO 110 J = 1,1000
                RESP(J) = RESP(J)*PHBIN
110        CONTINUE
C
C  The next step is to rebin each RESP into the pulse height bins of
C  the response matrix.  RESP is a pulse-height integrated response
C  function evaluated at 1000 pulse-height bins.  Rebin into NBINS
C  bins and store the result in RBRESP.
C
           SUMRESP(1) = RESP(1)
           DO 120 K = 2,1000
                SUMRESP(K) = SUMRESP(K-1) + RESP(K)
120        CONTINUE
           SUMLO = TERP(PHLIMS(1),RESPPH,SUMRESP)
           DO 130 J = 2,NBINS+1
```

```
               SUMHI = TERP(PHLIMS(J),RESPPH,SUMRESP)
               RBRESP(J-1) = SUMHI - SUMLO
               SUMLO = SUMHI
130         CONTINUE
C
C  Load all NRESP properly binned, pulse-height integrated response
C  functions into the 2-D array SUBRESP.
C
          DO 140 K = 1,NBINS
               SUBRESP(I,K) = RBRESP(K)
140         CONTINUE
150    CONTINUE
       J = 0
C
C  There are three response functions per response matrix energy bin.
C  Convert these three response functions into one response function
C  per energy bin by averaging the three.  The result is FINRESP--
C  a 2-D array which is the response matrix.  FINRESP(I',J') is the
C  probability that a gamma ray in the energy bin I' produces a count
C  in the pulse-height bin J'.
C
       DO 170 L = 1,NRESP,3
            J = J + 1
            DO 160 K = 1,NBINS
                 FINRESP(J,K) = (SUBRESP(L,K) + SUBRESP(L+1,K)
     1                         + SUBRESP(L+2,K))/3.0
160         CONTINUE
170    CONTINUE
C
C  Output the response matrix to a file for ease of reading--BGRESP.OUT.
C
       OPEN(UNIT=2,FILE='BGRESP.OUT',STATUS='[illegible]')
       WRITE(2,180)
180    FORMAT(T10,'**ADOPTED BINS FOR THE RESPONSE MATRIX**'/)
       WRITE(2,190)
190    FORMAT(1X,54('-')/1X,54('-'))
       WRITE(2,200)
200    FORMAT(T6,'Bin',T15,'Bin',T27,'Bin',T44,'Bin',
     1 /T5,'Number',T13,'Centroid',T26,'Width',T43,
     2 'Limits',/T14,'(keV)',T26,'(keV)',T43,'(keV)')
       WRITE(2,190)
       DO 220 I = 1,NBINS
            BINC = (PHLIMS(I) + PHLIMS(I+1))/2.0
            BINW = PHLIMS(I+1) - PHLIMS(I)
            WRITE(2,210) I,BINC,BINW,PHLIMS(I),PHLIMS(I+1)
210         FORMAT(T6,I3,T13,F8.2,T25,F7.2,T36,F8.2,
     1              1X,'-',1X,F8.2)
220    CONTINUE
       WRITE(2,190)
       WRITE(2,230) TYPE
230    FORMAT(//T10,'Response Matrix For ',A17,' BGO Inorganic',
     1 ' Scintillator'/T19,'Column = Gamma-ray Energy, Row = ',
     2 'Pulse Height')
       WRITE(2,240)
```

```
240   FORMAT(//)
      DO 270 I = 1,NBINS,7
          MM = NBINS - I
          IF(MM.GT.6) THEN
              NN = 6
          ELSE
              NN = MM
          ENDIF
          DO 260 J = 1,NBINS
              WRITE(2,250) (FINRESP(I+K,J),K=0,NN)
250           FORMAT(1X,7(2X,1PE9.3))
260       CONTINUE
          WRITE(2,240)
270   CONTINUE
      CLOSE(UNIT=2)
C
C  Output the file in a succint format for input to an unfolding code.
C  Note that the highest energy response of the matrix, I=NBINS, is not
C  printed to this file.  The unfolding code intended for use of this
C  file, BGO.MAT, does not need the highest energy response of the
C  matrix.  If a different unfolding code is used, modification of
C  this file must be made.
C
      OPEN(UNIT=2,FILE='BGO.MAT',STATUS='NEW')
      WRITE(2,280) ((FINRESP(I,J),I=1,NBINS-1),J=1,NBINS)
280   FORMAT(8(1X,1PE9.3))
      CLOSE(UNIT=2)
C
C  Output the number of bins in the response matrix and the pulse-
C  height values of these bins to the file BGLIMS.DAT.  This file
C  is used by an unfolding code.  The pulse-height values in this
C  file are in MeV and not keV.
C
      OPEN(UNIT=2,FILE='BGLIMS.DAT',STATUS='NEW')
      WRITE(2,290) NBINS
290   FORMAT(I5)
      WRITE(2,300) (PHLIMS(I)/1000.0,I=1,NBINS+1)
300   FORMAT(5(6X,F7.4))
      CLOSE(UNIT=2)
      END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      SUBROUTINE BG1RESP(ENRESP)
C
C  Subroutine BG1RESP calculates an absolute differential efficiency
C  curve for gamma rays for a 2.54-cm x 2.54-cm BGO inorganic scintillator
C  for any specified gamma ray energy.  This curve is what is normally
C  called the response function.  The response function is generated
C  from an interpolation scheme using 12 data files which contain
C  experimental response functions for monoenergetic fluxes of gamma
C  rays incident on the detector (source-to-detector distance equals
C  one meter) with the following energies:  123.6, 279.2, 514.0, 661.6,
C  834.8, 1115.5, 1274.5, 1836.0, 2754.0, 4440.0, 6140.0, and
```

```
C 11670.0 keV.  The calculated response function is stored in the array
C RESP.  RESP(I') is the probability per unit pulse height (in keV)
C for one gamma ray of the specified energy to produce a count in the
C pulse-height bin I'.
C
C Subroutine Summary:
C
C Other Subroutines Used:
C
C     None.
C
C Functions Used:
C
C     None.
C
C Common Blocks Used:
C
C     ARRAYS   == Contains 8 1-D arrays shared between this
C                 subroutine and the subroutine BG3RESP which
C                 is identical to this subroutine except for the
C                 data statements.
C     DATARAYS == Contains 2 1-D arrays shared between this
C                 subroutine and the main program and subroutine
C                 BG3RESP.
C     PARM     == Contains 3 parameters shared between this
C                 subroutine and the main program and subroutine
C                 BG3RESP.
C
C Local Variables Used:
C
C     BGNORM   == Real 1-D 12 element array.  Contains the
C                 normalization # for each experimental response
C                 function data file in order to convert the data
C                 from relative counts to counts per incident
C                 gamma ray.
C     CTSHI    == Real 1-D 1024 element array.  In Common Block
C                 ARRAYS.  Same as array CTSLO except that it
C                 contains the information for the gamma-ray
C                 energy which is the closest, but greater than
C                 or equal to, the specified energy.
C     CTSLO    == Real 1-D 1024 element array.  In Common Block
C                 ARRAYS.  Contains the experimental response
C                 function (pulse-height integrated--counts/
C                 gamma ray) for the gamma-ray energy which
C                 is the closest, but less than or equal to,
C                 the energy passed to the subroutine for
C                 response function computation.
C     DELTAHI  == Real variable.  The absolute energy distance
C                 between EHI and ENRESP.
C     DELTALO  == Real variable.  The absolute energy distance
C                 between ELO and ENRESP.
C     DNDEHI   == Real 1-D 1024 element array.  In Common Block
C                 ARRAYS.  CTSHI divided by the experimental
C                 bin width and multiplied by BGNORM so that the
```

```
C                  experimental data is counts/keV/gamma ray.
C       DNDELO  == Real 1-D 1024 element array.  In Common Block
C                  ARRAYS.  CTSLO divided by the experimental
C                  bin width and multiplied by BGNORM.
C       DUMVAL  == Real variable.  A dummy variable.
C       EGAMMA  == Real 1-D 12 element array.  Contains the gamma-ray
C                  energy values for the 12 measured response functions.
C       EHI     == Real variable.  The energy of the data file which
C                  brackets ENRESP on the high side.
C       ELO     == Real variable.  The energy of the data file which
C                  brackets ENRESP on the low side.
C       ENRESP  == Real variable.  Passed to the subroutine through
C                  the call.  This is the gamma-ray energy for
C                  computation of a response function.
C       GFILE   == Character 1-D 12 element array.  Contains the
C                  file names of the experimental response function
C                  data files.
C       GSLOPE  == Real 1-D 12 element array.  Contains the slope
C                  in keV/Channel # for the pulse-height axis
C                  calibration for each experimental response
C                  function data file.
C       GYINT   == Real 1-D 12 element array.  Contains the intercept
C                  in keV for the pulse-height axis calibration
C                  for each experimental response function data file.
C       INC     == Integer variable.  A counter.
C       JHI1    == Integer variable.  A counter.
C       JHI2    == Integer variable.  A counter.
C       JLO1    == Integer variable.  A counter.
C       JLO2    == Integer variable.  A counter.
C       MHI     == Integer variable.  A counter.
C       MLO     == Integer variable.  A counter.
C       PHBIN   == Real variable.  In Common Block PARM.  See
C                  definition in the main program.
C       PHCUT   == Real 1-D 12 element array.  One value for each
C                  experimental response function data file.  Above
C                  this pulse-height value, the data file is shifted
C                  downward or upward in pulse height in order to
C                  match the pulse height of the response function
C                  for the gamma-ray energy ENRESP.
C       PHVAL   == Real variable.  A dummy variable for storing
C                  pulse-height values.
C       PINCHI  == Real variable.  The pulse-height increments needed
C                  to span the pulse-height range WIDTHHI.
C       PINCLO  == Real variable.  The pulse-height increments needed
C                  to span the pulse-height range WIDTHLO.
C       PLCUT   == Real 1-D 12 element array.  One value for each
C                  experimental response function data file.  Below
C                  this pulse-height value, the pulse-height axis
C                  of the data file is not modified.
C       PRESPH  == Real variable.  In Common Block PARM.  Same as
C                  PRANH in the main program.
C       PRESPL  == Real variable.  In Common Block PARM.  Same as
C                  PRANL in the main program.
C       PVALHI  == Real 1-D 1024 element array.  In Common Block
```

```
C                ARRAYS.  Contains the pulse-height bin values
C                (in keV) used to generate the response function
C                stored in RESPHI.
C     PVALLO  == Real 1-D 1024 element array.  In Common Block
C                ARRAYS.  Contains the pulse-height bin values
C                (in keV) used to generate the response function
C                stored in RESPLO.
C     RESP    == Real 1-D 1000 element array.  In Common Block
C                DATARAYS.  See definition in the main program.
C     RESPHI  == Real 1-D 1000 element array.  In Common Block
C                ARRAYS.  Contains the same response function
C                as stored in DNDEHI except it has been shifted
C                and compressed along the pulse-height axis
C                in order to simulate the pulse-height axis
C                of the response function for the gamma-ray energy
C                ENRESP.
C     RESPLO  == Real 1-D 1000 element array.  In Common Block
C                ARRAYS.  Contains the same response function
C                as stored in DNDELO except it has been shifted
C                and stretched along the pulse-height axis in
C                order to simulate the pulse-height axis of the
C                response function for the gamma-ray energy ENRESP.
C     RESPPH  == Real 1-D 1000 element array.  In Common Block
C                DATARAYS.  See definition in the main program.
C     WIDTHHI == Real variable.  Range of data in the experimental
C                response function data file for the gamma-ray energy
C                EHI which must be compressed.
C     WIDTHLO == Real variable.  Range of data in the experimental
C                response function data file for the gamma-ray energy
C                ELO which must be stretched.
C
C Set up the common blocks used between this subroutine and the main
C program and the subroutine BG3RESP.
C
      COMMON/PARM/PRESPL,PRESPH,PHBIN
      COMMON/DATARAYS/RESP(1000),RESPPH(1000)
      COMMON/ARRAYS/CTSLO(1024),CTSHI(1024),DNDELO(1024),
     1            DNDEHI(1024),PVALLO(1024),PVALHI(1024),
     2            RESPLO(1000),RESPHI(1000)
C
C Dimension the variables.
C
      CHARACTER GFILE(12)*11
      DIMENSION GSLOPE(12),GYINT(12),BGNORM(12),PLCUT(12),
     1        PHCUT(12),EGAMMA(12)
C
C Load the 12-element arrays with the necessary data.  GSLOPE, GYINT,
C PLCUT, PHCUT, and EGAMMA are energy or pulse-height values and
C the numbers all have units of keV.
C
      DATA GFILE/'BG124.D1T','BG279.D1T','BG514.D1T','BG662.D1T',
     1        'BG835.D1T','BG1116.D1T','BG1275.D1T',
     2        'BG1836.D1T','BG2754.D1T','BG4440.D1T',
     3        'BG6140.D1T','BG11670.D1T'/
```

```
      DATA GSLOPE/0.41803,0.42656,0.69732,0.8445,1.05882,1.4503,
     1          1.68546,2.2352,3.17087,4.90506,7.99978,13.75606/
      DATA GYINT/11.1417,12.0048,10.55345,8.559,8.7637,6.5508,
     1          12.45844,10.786,-6.14645,-3.7977,-23.565,-23.132/
      DATA BGNORM/2.26E-06,1.14E-05,3.096E-06,7.16E-07,1.49E-06,
     1          1.87E-06,1.01E-06,2.13E-06,1.59E-06,5.62E-07,
     2          8.94E-07,1.48E-05/
      DATA PLCUT/67.0,107.0,243.0,255.0,300.0,317.0,330.0,660.0,
     1          700.0,733.0,650.0,700.0/
      DATA PHCUT/90.0,200.0,342.0,425.0,573.0,803.0,960.0,
     1          1158.0,1560.0,3167.0,4860.0,9583.0/
      DATA EGAMMA/123.6,279.2,514.0,661.6,834.8,1115.5,1274.5,
     1          1836.0,2754.0,4440.0,6140.0,11670.0/
C
C  This loop determines which experimental response function data files
C  bracket (in energy) the response function which is to be computed
C  at the gamma-ray energy ENRESP.
C
      DO 10 I = 2,12
          IF(ENRESP.LE.EGAMMA(I)) THEN
              ELO = EGAMMA(I-1)
              EHI = EGAMMA(I)
              MLO = I - 1
              MHI = I
              GO TO 15
          ENDIF
10    CONTINUE
15    CONTINUE
C
C  Open the two experimental response function data files which bracket
C  the response function to be calculated and read the data--the data
C  is such that it has the units of counts.  That is, each experimental
C  response function data file is pulse-height integrated (counts in a
C  bin) and is not normalized to the total number of gamma-rays incident
C  on the detector to produce the data file.
C
      OPEN(UNIT=2,FILE=GFILE(MLO),STATUS='OLD')
      OPEN(UNIT=3,FILE=GFILE(MHI),STATUS='OLD')
      READ(2,20) (CTSLO(I),I = 1,1024)
20    FORMAT(1X,6F13.2)
      READ(3,20) (CTSHI(I),I = 1,1024)
      CLOSE(UNIT=2)
      CLOSE(UNIT=3)
C
C  Normalize the experimental response function and divide each binned
C  count by the pulse-height bin width.  Thus, DNDELO and DNDEHI are
C  experimental absolute differential efficiency curves--i.e., they
C  have units of counts/unit pulse height (keV)/incident gamma ray.
C
      DO 30 I = 1,1024
          DNDELO(I) = CTSLO(I)*BGNORM(MLO)/GSLOPE(MLO)
          DNDEHI(I) = CTSHI(I)*BGNORM(MHI)/GSLOPE(MHI)
30    CONTINUE
C
```

```
C Determine the difference in energies between the response function
C to be calculated and the two response functions bracketing this
C response function and which will be used for the interpolation
C process.
C
      DELTALO = ENRESP - ELO
      DELTAHI = EHI - ENRESP
C
C The interpolation process is as follows:  First, the experimental
C response function which is for the gamma-ray energy just below or
C or equal to ENRESP is modified.  This modification is a modification
C of the pulse-height axis only.  The pulse-height axis is the original
C pulse-height axis below the value PLCUT, shifted upward by the amount
C DELTALO above the value PHCUT, and stretched in between to fill
C the remainder of the pulse-height axis.  A similar modification
C is made to the experimental response function which is for the
C gamma-ray energy just above or equal to ENRESP--the pulse-height axis
C is the original below the value PLCUT, shifted downward by the amount
C DELTAHI above the value PHCUT, and compressed in between to fill
C the remainder of the pulse-height axis.  Second, these two pulse-
C height modified response functions are then rebinned into 1000
C data points which are equally spaced about the pulse-height range
C specified by the user for the response matrix, i.e., the range
C PRESPL to PRESPH.  Third, these resulting two experimental response
C functions are then used to generate a response function at the
C gamma-ray energy ENRESP through a log-log interpolation along
C the y axis (counts/keV/gamma ray axis).
C
      INC = 0
C
C First step of the interpolation process.
C
      DO 40 I = 1,1024
           DUMVAL = GSLOPE(MLO)*I + GYINT(MLO)
           IF(DUMVAL.LE.PLCUT(MLO)) THEN
                PVALLO(I) = DUMVAL
                JLO1 = I
           ENDIF
           IF(DUMVAL.GE.PHCUT(MLO)) THEN
                INC = INC + 1
                PVALLO(I) = DUMVAL + DELTALO
           ENDIF
40    CONTINUE
      JLO2 = 1024 - INC + 1
      WIDTHLO = PVALLO(JLO2) - PVALLO(JLO1)
      PINCLO = WIDTHLO/(JLO2 - JLO1)
      DO 50 I = JLO1+1,JLO2-1
           PVALLO(I) = PVALLO(I-1) + PINCLO
50    CONTINUE
      INC = 0
      DO 60 I = 1,1024
           DUMVAL = GSLOPE(MHI)*I + GYINT(MHI)
           IF(DUMVAL.LE.PLCUT(MHI)) THEN
                PVALHI(I) = DUMVAL
```

```
                JHI1 = I
            ENDIF
            IF(DUMVAL.GE.PHCUT(MHI)) THEN
                INC = INC + 1
                PVALHI(I) = DUMVAL - DELTAHI
            ENDIF
60      CONTINUE
        JHI2 = 1024 - INC + 1
        WIDTHHI = PVALHI(JHI2) - PVALHI(JHI1)
        PINCHI = WIDTHHI/(JHI2 - JHI1)
        DO 70 I = JHI1+1,JHI2-1
            PVALHI(I) = PVALHI(I-1) + PINCHI
70      CONTINUE
C
C  Second step of the interpolation process.
C
        PHBIN = (PRESPH - PRESPL)/1000.0
        PHVAL = PRESPL - PHBIN/2.0
        DO 100 I = 1,1000
            PHVAL = PHVAL + PHBIN
            DO 80 J = 2,1024
                IF(PHVAL.LE.PVALLO(J)) THEN
                    RESPLO(I) = (DNDELO(J) - DNDELO(J-1))*
     1                          (PHVAL - PVALLO(J-1))/
     2                          (PVALLO(J) - PVALLO(J-1)) +
     3                          DNDELO(J-1)
                    IF(RESPLO(I).LT.0.0) RESPLO(I) = 0.0
                    GO TO 85
                ENDIF
80          CONTINUE
85          CONTINUE
            DO 90 J = 2,1024
                IF(PHVAL.LE.PVALHI(J)) THEN
                    RESPHI(I) = (DNDEHI(J) - DNDEHI(J-1))*
     1                          (PHVAL - PVALHI(J-1))/
     2                          (PVALHI(J) - PVALHI(J-1)) +
     3                          DNDEHI(J-1)
                    IF(RESPHI(I).LT.0.0) RESPHI(I) = 0.0
                    GO TO 95
                ENDIF
90          CONTINUE
95          CONTINUE
100     CONTINUE
C
C  Third and final step of the interpolation process.
C
        DO 110 I = 1,1000
            IF(RESPLO(I).EQ.0.0.OR.RESPHI(I).EQ.0.0) THEN
                RESP(I) = RESPLO(I)*(LOG(EHI) - LOG(ENRESP))
     1                    /(LOG(EHI) - LOG(ELO)) +
     2                    RESPHI(I)*(LOG(ENRESP) - LOG(ELO))
     3                    /(LOG(EHI) - LOG(ELO))
            ELSE
                RESP(I) = LOG(RESPLO(I))*(LOG(EHI) - LOG(ENRESP))
```

```
    1                 /(LOG(EHI) - LOG(ELO)) +
    2                 LOG(RESPHI(I))*(LOG(ENRESP) - LOG(ELO))
    3                 /(LOG(EHI) - LOG(ELO))
              RESP(I) = EXP(RESP(I))
          ENDIF
          RESPPH(I) = PHBIN*I + PRESPL - PHBIN/2.0
110   CONTINUE
      RETURN
      END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      SUBROUTINE BG3RESP(ENRESP)
C
C  Subroutine BG3RESP calculates an absolute differential efficiency
C  curve for gamma rays for a 7.62-cm x 7.62-cm BGO inorganic scintillator
C  for any specified gamma ray energy.  This subroutine is identical
C  to BG1RESP.  See the subroutine BG1RESP for documentation.
C
      COMMON/PARM/PRESPL,PRESPH,PHBIN
      COMMON/DATARAYS/RESP(1000),RESPPH(1000)
      COMMON/ARRAYS/CTSLO(1024),CTSHI(1024),DNDELO(1024),
     1             DNDEHI(1024),PVALLO(1024),PVALHI(1024),
     2             RESPLO(1000),RESPHI(1000)
      CHARACTER GFILE(12)*11
      DIMENSION GSLOPE(12),GYINT(12),BGNORM(12),PLCUT(12),
     1         PHCUT(12),EGAMMA(12)
      DATA GFILE/'BG124.D3T','BG279.D3T','BG514.D3T','BG662.D3T',
     1          'BG835.D3T','BG1116.D3T','BG1275.D3T',
     2          'BG1836.D3T','BG2754.D3T','BG4440.D3T',
     3          'BG6140.D3T','BG11670.D3T'/
      DATA GSLOPE/0.40024,0.3983,0.66238,0.90496,1.03376,1.4835,
     1           1.67676,2.34348,3.1788,5.31817,7.50905,13.29843/
      DATA GYINT/12.7296,9.7776,7.9805,5.43216,4.5459,0.11605,
     1          -6.0159,-17.099,-19.1988,-13.3556,-31.3755,
     2          -17.4087/
      DATA BGNORM/2.51E-07,1.92E-06,3.44E-07,7.96E-08,1.65E-07,
     1           2.08E-07,1.12E-07,2.36E-07,5.44E-07,1.66E-07,
     2           1.37E-07,2.55E-06
      DATA PLCUT/ 7[illegible]0,10[illegible].0,260.0,2[illegible]7.[illegible],330.0,3[illegible].0,340.0,601.0,
     1          601.0,750.0,700.0,700.0/
      DATA PHCUT/90.0,200.0,357.0,470.0,605.0,880.0,612.0,
     1          1129.0,1562.0,3233.0,4900.0,9370.0/
      DATA EGAMMA/123.6,279.2,514.0,661.6,834.8,1115.5,1274.5,
     1           1836.0,2754.0,4440.0,6140.0,11670.0/
      DO 10 I = 2,12
          IF(ENRESP.LE.EGAMMA(I)) THEN
              ELO = EGAMMA(I-1)
              EHI = EGAMMA(I)
              MLO = I - 1
              MHI = I
              GO TO 15
          ENDIF
10    CONTINUE
```

```
15    CONTINUE
      OPEN(UNIT=2,FILE=GFILE(MLO),STATUS='OLD')
      OPEN(UNIT=3,FILE=GFILE(MHI),STATUS='OLD')
      READ(2,20) (CTSLO(I),I = 1,1024)
20    FORMAT(1X,6F13.2)
      READ(3,20) (CTSHI(I),I = 1,1024)
      CLOSE(UNIT=2)
      CLOSE(UNIT=3)
      DO 30 I = 1,1024
          DNDELO(I) = CTSLO(I)*BGNORM(MLO)/GSLOPE(MLO)
          DNDEHI(I) = CTSHI(I)*BGNORM(MHI)/GSLOPE(MHI)
30    CONTINUE
      DELTALO = ENRESP - ELO
      DELTAHI = EHI - ENRESP
      INC = 0
      DO 40 I = 1,1024
          DUMVAL = GSLOPE(MLO)*I + GYINT(MLO)
          IF(DUMVAL.LE.PLCUT(MLO)) THEN
              PVALLO(I) = DUMVAL
              JLO1 = I
          ENDIF
          IF(DUMVAL.GE.PHCUT(MLO)) THEN
              INC = INC + 1
              PVALLO(I) = DUMVAL + DELTALO
          ENDIF
40    CONTINUE
      JLO2 = 1024 - INC + 1
      WIDTHLO = PVALLO(JLO2) - PVALLO(JLO1)
      PINCLO = WIDTHLO/(JLO2 - JLO1)
      DO 50 I = JLO1+1,JLO2-1
          PVALLO(I) = PVALLO(I-1) + PINCLO
50    CONTINUE
      INC = 0
      DO 60 I = 1,1024
          DUMVAL = GSLOPE(MHI)*I + GYINT(MHI)
          IF(DUMVAL.LE.PLCUT(MHI)) THEN
              PVALHI(I) = DUMVAL
              JHI1 = I
          ENDIF
          IF(DUMVAL.GE.PHCUT(MHI)) THEN
              INC = INC + 1
              PVALHI(I) = DUMVAL - DELTAHI
          ENDIF
60    CONTINUE
      JHI2 = 1024 - INC + 1
      WIDTHHI = PVALHI(JHI2) - PVALHI(JHI1)
      PINCHI = WIDTHHI/(JHI2 - JHI1)
      DO 70 I = JHI1+1,JHI2-1
          PVALHI(I) = PVALHI(I-1) + PINCHI
70    CONTINUE
      PHBIN = (PRESPH - PRESPL)/1000.0
      PHVAL = PRESPL - PHBIN/2.0
      DO 100 I = 1,1000
          PHVAL = PHVAL + PHBIN
```

```
          DO 80 J = 2,1024
               IF(PHVAL.LE.PVALLO(J)) THEN
                    RESPLO(I) = (DNDELO(J) - DNDELO(J-1))*
     1                          (PHVAL - PVALLO(J-1))/
     2                          (PVALLO(J) - PVALLO(J-1)) +
     3                          DNDELO(J-1)
                    IF(RESPLO(I).LT.0.0) RESPLO(I) = 0.0
                    GO TO 85
               ENDIF
80        CONTINUE
85        CONTINUE
          DO 90 J = 2,1024
               IF(PHVAL.LE.PVALHI(J)) THEN
                    RESPHI(I) = (DNDEHI(J) - DNDEHI(J-1))*
     1                          (PHVAL - PVALHI(J-1))/
     2                          (PVALHI(J) - PVALHI(J-1)) +
     3                          DNDEHI(J-1)
                    IF(RESPHI(I).LT.0.0) RESPHI(I) = 0.0
                    GO TO 95
               ENDIF
90        CONTINUE
95        CONTINUE
100   CONTINUE
      DO 110 I = 1,1000
          IF(RESPLO(I).EQ.0.0.OR.RESPHI(I).EQ.0.0) THEN
               RESP(I) = RESPLO(I)*(LOG(EHI) - LOG(ENRESP))
     1                  /(LOG(EHI) - LOG(ELO)) +
     2                  RESPHI(I)*(LOG(ENRESP) - LOG(ELO))
     3                  /(LOG(EHI) - LOG(ELO))
          ELSE
               RESP(I) = LOG(RESPLO(I))*(LOG(EHI) - LOG(ENRESP))
     1                  /(LOG(EHI) - LOG(ELO)) +
     2                  LOG(RESPHI(I))*(LOG(ENRESP) - LOG(ELO))
     3                  /(LOG(EHI) - LOG(ELO))
               RESP(I) = EXP(RESP(I))
          ENDIF
          RESPPH(I) = PHBIN*I + PRESPL - PHBIN/2.0
110   CONTINUE
      RETURN
      END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      FUNCTION TERP(PHVAL,PHITE,COUNTS)
C
C  This function is a simple interpolation routine and is self-explanatory.
C
      DIMENSION PHITE(1000),COUNTS(1000)
      IF(PHVAL.LE.PHITE(1)) THEN
          TERP = COUNTS(1)
          RETURN
      ENDIF
      IF(PHVAL.GT.PHITE(1000)) THEN
          TERP = COUNTS(1000)
```

```
          RETURN
      ENDIF
      DO 10 I = 2,1000
          IF(PHVAL.LE.PHITE(I)) THEN
              TERP = (PHVAL - PHITE(I-1))/(PHITE(I) - PHITE(I-1))
     1                *(COUNTS(I) - COUNTS(I-1)) + COUNTS(I-1)
              RETURN
          ENDIF
   10 CONTINUE
      END
```